

4. Empirical Study

To test the approach we proposed, three sets of data of foreign exchange rate are chosen as our empirical examples. They are the foreign exchange rate of EUR(Europe), CAD(Canada) and GBP(U.K.) with respect to USD(U.S.A). The reason why we select the three sets of data is that they are different time series in structure. For demonstrating that our approach can be generally applied, the data analyzed here was chosen arbitrarily from January 1, 2003, the beginning of this year, to July 25, 2003, when this paper is complete, instead of choosing a certain period of time deliberately.

Each set of data is given a total number of observations of 144. The purpose of this study is to detect of change periods in a time series of real world data and test the performance of our approach. Figure 4.1, Figure 4.2 and Figure 4.3 are the plot of the foreign exchange rates of EUR against USD, USD against CAD and GBP against USD, respectively.

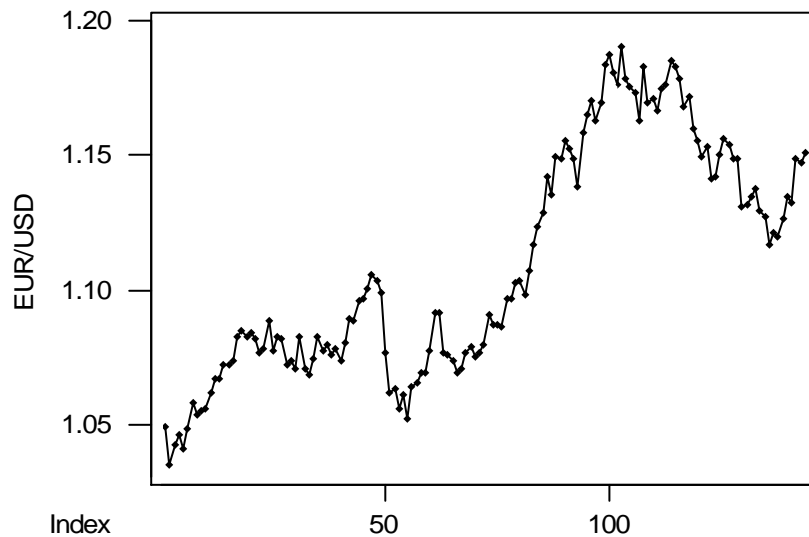


Figure (4.1) The exchange rate of EUR against USD

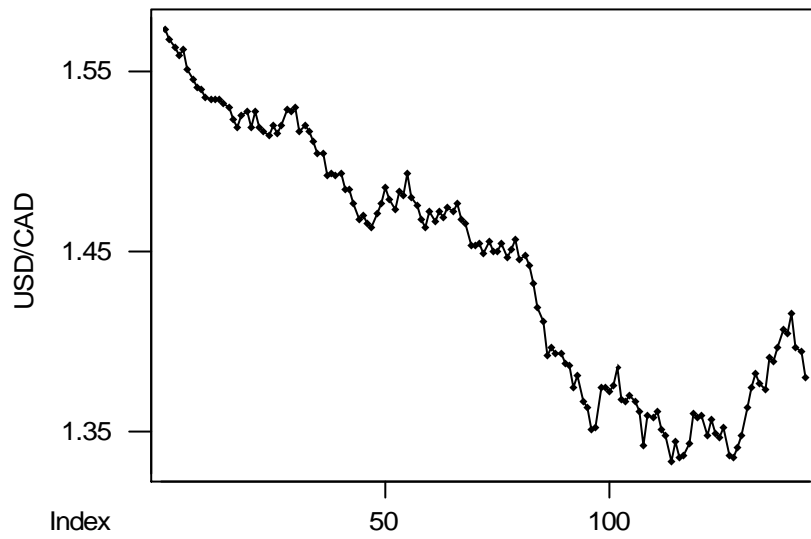


Figure (4.2) The exchange rate of USD against CAD

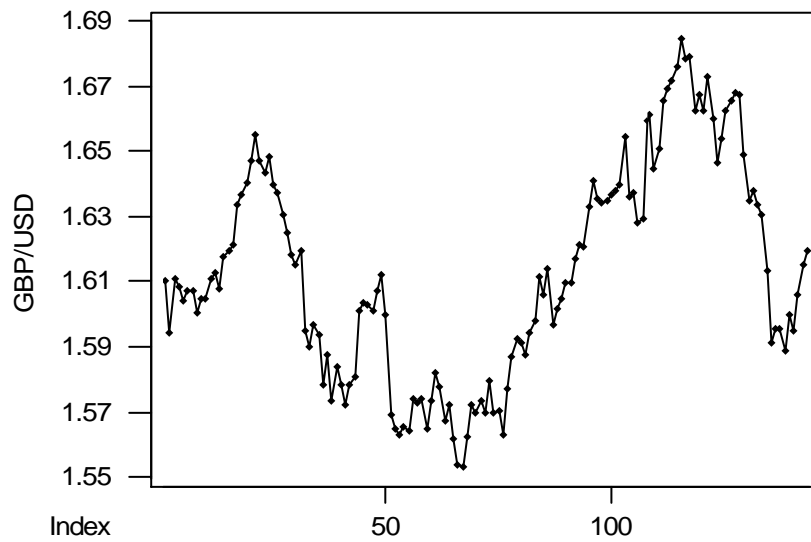


Figure (4.3) The exchange rate of GBP against USD

According to section 2, we have to make the first difference series $\{Y_t\}$ of these data first. Then we transforming $\{Y_t\}$ into a fuzzy time series with the linguistic values set: $U = \{L_i, i = 1, 2, 3, 4, 5\} = \{fall\ sharp, fall, unchange, rise, rise\ sharp\}$ and the membership function are shown in Figure (4.4) to (4.6).

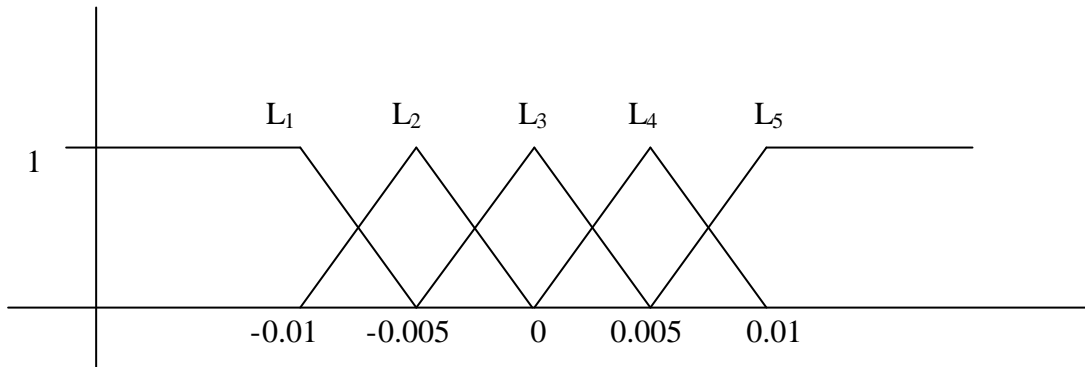


Figure (4.4) Membership function of the exchange rate of EUR against USD

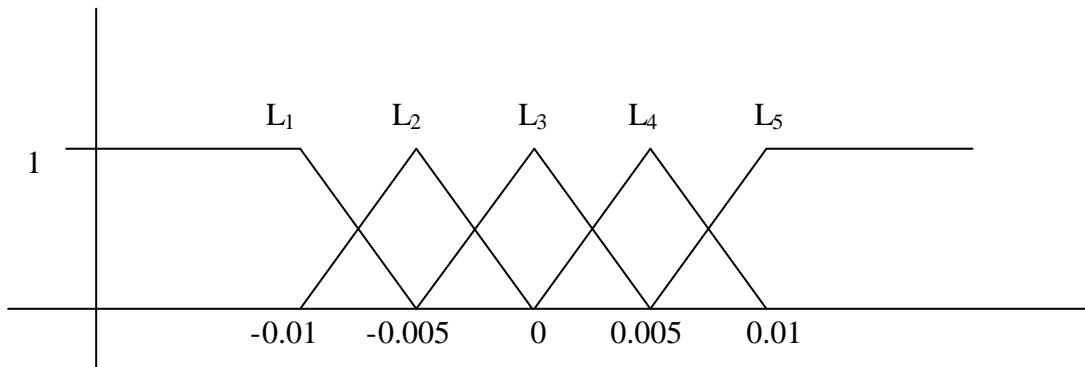


Figure (4.5) Membership function of the exchange rate of USD against CAD

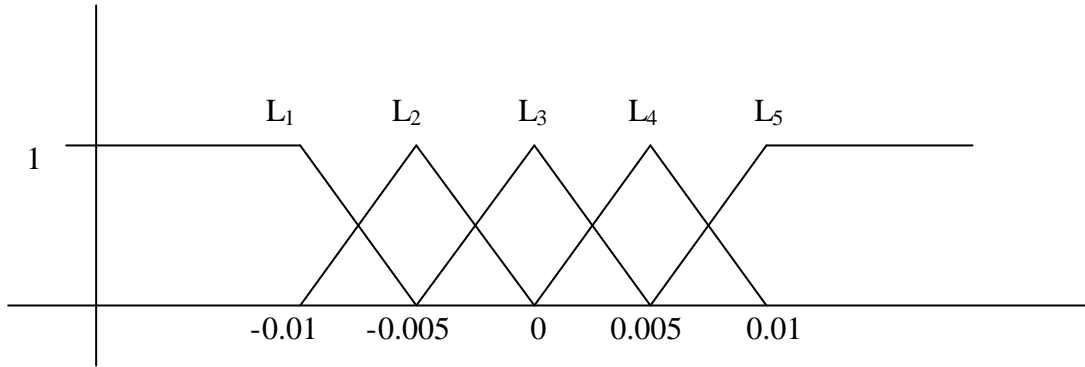


Figure (4.6) Membership function of the exchange rate of GBP against USD

In these three time series, we set the fuzzy weights a_1, a_2, a_3, a_4, a_5 to be, respectively, -3, -1, 0, 1, 3. The degree of the change period detection sequence and the h in definition 2.6 are set to be 10 and 5. Following the process we proposed, we can find change periods of the time series of the foreign exchange rate of EUR against USD $T_1 = \{ t_{15}, t_{16}, \dots, t_{35} \}$, $T_2 = \{ t_{44}, t_{45}, \dots, t_{47} \}$, $T_3 = \{ t_{53}, t_{54}, \dots, t_{57} \}$, $T_4 = \{ t_{60}, t_{61}, \dots, t_{70} \}$, $T_5 = \{ t_{98}, t_{99}, \dots, t_{102} \}$, $T_6 = \{ t_{106}, t_{107}, \dots, t_{115} \}$, $T_7 = \{ t_{121}, t_{122}, \dots, t_{129} \}$ and $T_8 = \{ t_{134}, t_{135}, \dots, t_{138} \}$. We show the partial time series on T_1 to T_8 graphically in Figure 4.7.

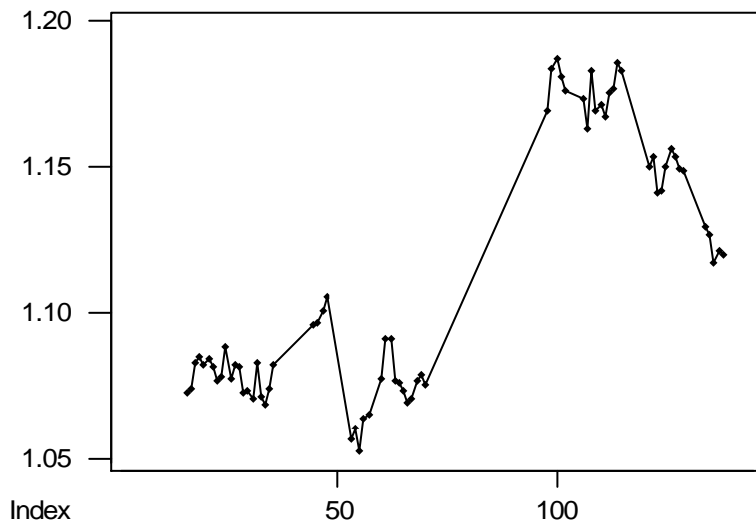


Figure (4.7) Partial time series at change periods of exchange rate of EUR/USD

In the foreign exchange rate of USD against CAD, we found change periods, $T_1 = \{t_{13}, t_{14}, \dots, t_{31}\}$, $T_2 = \{t_{43}, t_{44}, \dots, t_{48}\}$, $T_3 = \{t_{50}, t_{51}, \dots, t_{65}\}$, $T_4 = \{t_{72}, t_{73}, \dots, t_{76}\}$, $T_5 = \{t_{95}, t_{96}, \dots, t_{106}\}$, $T_6 = \{t_{112}, t_{113}, \dots, t_{125}\}$, $T_7 = \{t_{137}, t_{138}, t_{139}\}$, at which the partial time series are plotted in Figure (4.8).

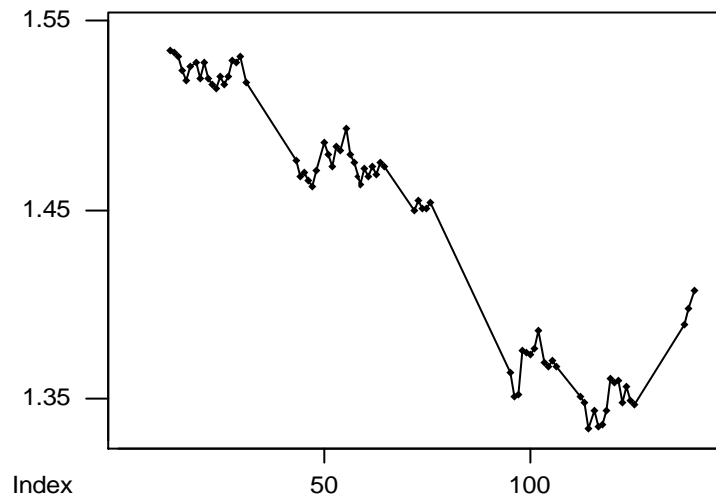


Figure (4.8) Partial time series at change periods of exchange rate of USD/CAD

In the foreign exchange rate of GBP against USD, the change periods were detected at $T_1 = \{t_5, t_6, \dots, t_{10}\}$, $T_2 = \{t_{19}, t_{20}, \dots, t_{23}\}$, $T_3 = \{t_{33}, t_{34}, \dots, t_{41}\}$, $T_4 = \{t_{43}, t_{44}, \dots, t_{53}\}$, $T_5 = \{t_{54}, t_{55}, \dots, t_{59}\}$, $T_6 = \{t_{62}, t_{63}, \dots, t_{70}\}$, $T_7 = \{t_{98}, t_{99}, \dots, t_{110}\}$, $T_8 = \{t_{114}, t_{115}, \dots, t_{125}\}$, $T_9 = \{t_{136}, t_{137}, t_{138}\}$. Figure (4.9) shows the partial time series at these change periods.

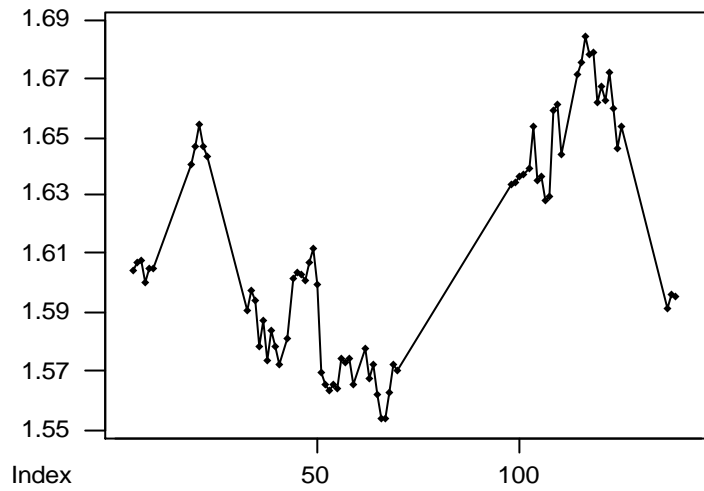


Figure (4.8) Partial time series at change periods of exchange rate of GBP/USD

Comparing Figure (4.1) with Figure (4.7), Figure (4.2) with Figure (4.8) and Figure (4.3) with Figure (4.9), we can see that the result is satisfied. The change periods that we detected in these three foreign exchange rates contains almost all the change periods that we objectively recognized. Since the data are selected arbitrarily in both duration of time and currencies, the performance of our approach shows that our proposed procedure can be applied in general models. Different kinds of desired change periods can be obtained by adjusting the parameters of the procedure.