Abstract

In this thesis, we shall focus on two independent topics in combinatorics: One is the constructions of Hadamard matrices and the other is the studies of forests. In the first topic, we obtain two main results. First, we use the Sylvester's approach to construct another Hadamard matrix, named a J_m-Hadamard matrix, from a given one. Consequently, we use permutations on S_m to generate other $2^m m! - 1$ Hadamard matrices from the constructed J_m-Hadamard matrices. We also introduce the J_m-class CJ_m for $m = 2$ or $m = 4k$ and study the problem whether $CJ_{n'} \subseteq CJ_n$, where $n|n'$; our initial result shows that $CJ_8 \subsetneqq CJ_4 \subsetneqq CJ_2$. Secondly, for $t \geq 4$, given any Hadamard matrices of orders $4m_1, 4m_2, ..., 4m_t$, we wish to yield a new Hadamard matrix of order $2^k m_1 m_2 \cdots m_t$ such that the exponent k is as small as possible. We derive some upper bound, which is better than those obtained by Craigen and de Launey, of the minimum exponent. In the second topic, we devote ourself to three themes. First, we generalize some results of plane trees, including Shapiro's result, odd or even number of leaves and Catalan-like equalities, to plane forests. Secondly, we present a neat bijective proof of Chung-Feller Theorem and obtain related results. Finally, we focus on graceful labellings of some $n-$caterpillars. In particular, we can use Latin squares to yield graceful labellings of 2^n-caterpillars.

Key Words: Kronecker product, Sylvester-Hadamard matrices, J_m-Hadamard matrices, J_m-classes, Orthogonal pairs, Weighing matrices, Minimum Exponent, Plane forests, Catalan numbers, Motzkin numbers, Riordan numbers, Narayana numbers, Dyck paths, Motzkin paths, Chung-Feller Theorem, Graceful labellings, n-Caterpillars ,Latin squares.