1 Introduction

A *mixed hypergraph* is a triple $\mathcal{H} = (X, \mathcal{C}, \mathcal{D})$, where X is the vertex set and each of $\mathcal{C}, \mathcal{D}$ is a list of subsets of X, the *$\mathcal{C}$-edges* and *$\mathcal{D}$-edges*, respectively, and we assume that all $\mathcal{C}$-edges and $\mathcal{D}$-edges have at least two elements. A *proper k-coloring* of a mixed hypergraph is a function, c, from the vertex set to a set of k colors so that each $\mathcal{C}$-edge has two vertices with a common color and each $\mathcal{D}$-edge has two vertices with distinct colors. A mixed hypergraph is *k-colorable* if it has a proper coloring with at most k colors. A *strict k-coloring* is a proper k-coloring using all k colors, it means that the function, c, is onto. In a colorable mixed hypergraph $\mathcal{H}$, the maximum(minimum) number of colors over all strict k-colorings is called the *upper(lower) chromatic number* of $\mathcal{H}$ and is denoted by $\bar{\chi}(\mathcal{H})(\chi(\mathcal{H}))$.

We use n to denote $|X|$ for the mixed hypergraph. Every proper k-coloring induces a partition of vertex set into color classes. Such partition, $\{X_1, X_2, \ldots, X_k\}$, is called a *feasible partition* with respect to the coloring. The number of feasible partitions into k colors is denoted by r_k. The vector $R(\mathcal{H}) = (r_1, \ldots, r_n)$ is called the *chromatic spectrum* of $\mathcal{H}$. The set of values k such that $\mathcal{H}$ has a strict k-coloring is the *feasible set* of $\mathcal{H}$, written by $S(\mathcal{H})$; this is the set of indices i such that $r_i > 0$. A mixed hypergraph is *uniquely colorable* if it allows exactly one feasible partition of the vertex set X into color classes. Therefore, $\chi(\mathcal{H}) = \bar{\chi}(\mathcal{H}) = \chi,\ r_\chi(\mathcal{H}) = 1$, and $R(\mathcal{H}) = (0, \ldots, 0, 1, 0, \ldots, 0)$. If $\mathcal{H}$ has no multiple edges and all its edges are of size r, then $\mathcal{H}$ is called an *r-uniform* hypergraph.

In chapter 2, we show that the size of vertex set of (l, m)-uniform mixed hypergraphs with unique coloring is more than $(l-1)(m-1)+1$. In chapter 2, we come up a way to construct uniquely colorable (l, m)-uniform mixed hypergraphs. But we discover that this way needs a lot of $\mathcal{C}$-edges and $\mathcal{D}$-edges. Following chapter 2, we will introduce two ways to construct uniquely colorable mixed hypergraphs. In chapter 3, we first consider r-uniform $\mathcal{C}$-hypergraphs and $\mathcal{D}$-hypergraphs. And we construct uniquely colorable r-uniform $\mathcal{C}$-hypergraphs and $\mathcal{D}$-hypergraphs. In chapter 4, we construct uniquely colorable (l, m)-uniform mixed hypergraphs by reducing $\mathcal{C}$-edges or $\mathcal{D}$-edges.