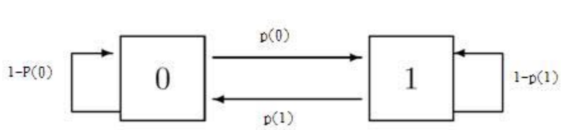
Chapter 3

Markov Modulated Bernoulli Process of Arrivals of VBR Cells

We assume VBR traffic is modeled by MMBP. We give a short review of MMBP [11]. Let $p(s_k)$ denote the probability that s_k changes at the end of the kth frame given that at the beginning of the kth frame it is in status s_k. Since the VBR cell arrival process is an MMBP, $\{s_k = 0, 1\}$ is a two state Markov chain. Then described is the diagram in Figure 3.1.

Figure 3.1: An MMBP process



It can be written from the definition of $p(s_k)$ that

$$Pr\{s_{k+1} = 1 | s_k = 0\} = p(0); \qquad Pr\{s_{k+1} = 0 | s_k = 0\} = 1 - p(0);$$

$$Pr\{s_{k+1} = 0 | s_k = 1\} = p(1); \qquad Pr\{s_{k+1} = 1 | s_k = 1\} = 1 - p(1).$$

Let $y(s_k)$ be the stationary probability of MMBP of VBR in status s_k, $s_k = 0$ or 1. Then the stationary probability of MMBP is

$$y(0) = \frac{p(1)}{p(0) + p(1)}, \quad y(1) = \frac{p(0)}{p(0) + p(1)}. \tag{3.1}$$

where it satisfies,

$$(y(0), y(1)) \begin{bmatrix} 1 - p(0) & p(0) \\ p(1) & 1 - p(1) \end{bmatrix} = (y(0), y(1)), \tag{3.2}$$

$$y(0) + y(1) = 1. \tag{3.3}$$

The results of (3.1) is computed by (3.2) and (3.3). Assume the transmission on line has a constant rate. Notice there is only a cell difference of the buffer size of VBR which we consider it implicitly in the modeling. Let $v(s_k)$ represent the probability of VBR arrives in one slot in status s_k. We can derive the probability of VBR occurring at one slot that is

$$\begin{aligned} v(0) &= Pr\{\text{VBR arrives in one slot in status } s_k = 0\} \\ &= Pr\{\text{VBR arrives in one slot} \mid s_k = 0\} Pr\{s_k = 0\} \\ &= r(0)y(0), \end{aligned}$$

similarly,

$$\begin{aligned} v(1) &= Pr\{\text{VBR arrives in one slot in status } s_k = 1\} \\ &= Pr\{\text{VBR arrives in one slot} \mid s_k = 1\} Pr\{s_k = 1\} \\ &= r(1)y(1). \end{aligned}$$

Let $\phi(s_k)$ be the probability of the status is in s_k given that a VBR cell arrives. We have

$$\begin{aligned} \phi(0) &= Pr\{\text{the status is in } s_k = 0, \text{ given that a VBR cell arrives}\} \\ &= \frac{Pr\{\text{a VBR cell arrives in a slot and is in } s_k = 0\}}{Pr\{\text{a VBR cell arrives in a slot}\}} \\ &= \frac{r(0)y(0)}{r(0)y(0) + r(1)y(1)} \\ &= \frac{r(0)p(1)}{r(0)p(1) + r(1)p(0)}. \end{aligned}$$

Similarly, it gives

$$\begin{aligned}
\phi(1) &= Pr\{\text{the status is in } s_k = 1, \text{ given that VBR cell arrives}\} \\
&= \frac{Pr\{\text{VBR cell arrives in a slot and is in } s_k = 1\}}{Pr\{\text{VBR cell arrives in a slot}\}} \\
&= \frac{r(1)y(1)}{r(0)y(0) + r(1)y(1)} \\
&= \frac{r(1)p(0)}{r(0)p(1) + r(1)p(0)}.
\end{aligned}$$

Next, we study inter-arrival time of VBR cells. Let N be the inter-arrival time of a VBR cell, the time interval to next arrival and N_{s_k} be that given the MMBP is in s_k. We have

$$N_0 = \begin{cases} 1, & \text{with probability } (1-p(0))r(0) + p(0)r(1), \\ 1 + N_0, & \text{with probability } (1-p(0))(1-r(0)), \\ 1 + N_1, & \text{with probability } p(0)(1-r(1)), \end{cases}$$

and

$$N_1 = \begin{cases} 1, & \text{with probability } (1-p(1))r(1) + p(1)r(0), \\ 1 + N_1, & \text{with probability } (1-p(1))(1-r(1)), \\ 1 + N_0, & \text{with probability } p(1)(1-r(0)). \end{cases}$$

After some manipulations, the z-transform of N_0 and N_1 are

$$N_0(z) = \frac{(1-r(1))r(0)(p(0)+p(1)-1)z^2 + [(1-p(0))r(0) + p(0)r(1)]\, z}{(1-r(1))(1-r(0))(3-p(0)-p(1))z^2 - [(1-p(1))(1-r(1)) + (1-p(0))(1-r(0))]\, z + 1}$$

and

$$N_1(z) = \frac{(1-r(0))r(1)(p(0)+p(1)-1)z^2 + [(1-p(1))r(1) + p(1)r(0)]\, z}{(1-r(1))(1-r(0))(3-p(0)-p(1))z^2 - [(1-p(1))(1-r(1)) + (1-p(0))(1-r(0))]\, z + 1}$$

Because

$$Pr\{N = n\} = Pr\{N = n | s = 0\} Pr\{s = 0\} + Pr\{N = n | s = 1\} Pr\{s = 1\},$$

it gives

$$\sum_{n=1}^{\infty} Pr\{N = n\} z^n = \sum_{n=1}^{\infty} Pr\{N = n | s = 0\} z^n Pr\{s = 0\} + \sum_{n=1}^{\infty} Pr\{N = n | s = 1\} z^n Pr\{s = 1\},$$

and

$$\begin{aligned} N(z) &= N_0(z)Pr\{s=0\} + N_1(z)Pr\{s=1\} \\ &= N_0(z)\phi(0) + N_1(z)\phi(1) \\ &= \frac{b_4 z^2 + b_3 z}{b_2 z^2 + b_1 z + b_0} \end{aligned}$$

where

$$\begin{aligned} b_0 &= r(0)p(1) + r(1)p(0), \\ b_1 &= -\left[(1-p(1))(1-r(1)) + (1-p(0))(1-r(0))\right]\left(r(0)p(1) + r(1)p(0)\right), \\ b_2 &= (1-r(1))(1-r(0))(3-p(0)-p(1))(r(0)p(1)+r(1)p(0)), \\ b_3 &= r^2(0)p(1) + r^2(1)p(0) - [r^2(0)+r^2(1)]p(0)p(1) + 2p(1)p(0)r(1)r(0), \\ b_4 &= (p(0)+p(1)-1)[r^2(0)p(1)(1-r(1)) + r^2(1)p(0)(1-r(0))]. \end{aligned}$$

Thus, we have

$$E[N] = \left.\frac{dN(z)}{dz}\right|_{z=1} = \frac{p(0)+p(1)}{p(1)r(0)+p(0)r(1)}$$

and

$$E[N^2] = \left.\frac{d^2N(z)}{dz^2}\right|_{z=1} + \left.\frac{dN(z)}{dz}\right|_{z=1}.$$

Then the squared coefficient of variation is

$$c_{sq}^2 = \frac{2[(p(0)+p(1))^2 + (p(0)r(0)+p(1)r(1))(1-p(0)-p(1))]}{E[N](p(0)+p(1))[p(1)r(0)+p(0)r(1)+r(0)r(1)(1-p(0)-p(1))]} - \frac{1}{E[N]} - 1.$$

Since

$$c_{sq}^2 = \frac{V[N]}{E^2[N]},$$

then we have

$$V[N] = c_{sq}^2 E^2[N].$$