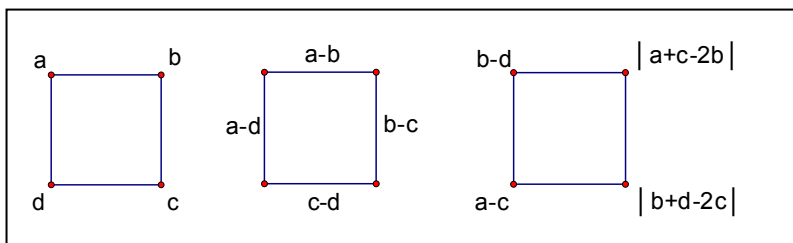


# 附錄一

四個頂點數字都不相同的情形：

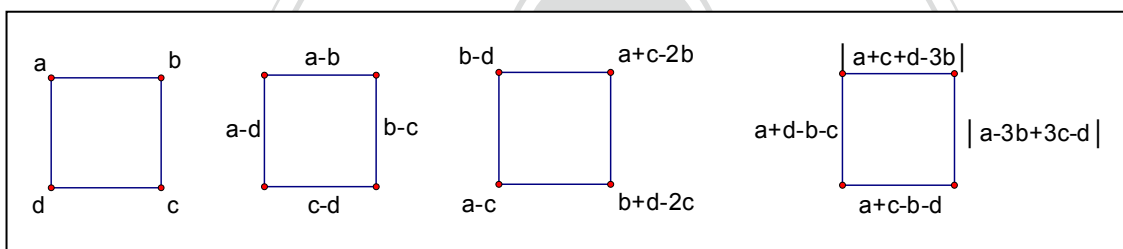
設此四數為  $a, b, c, d$ ，且  $a > b > c > d$ ，那麼不失一般性，用此四數排列迪菲方塊 (DIFFY BOX) 有下列三種組合： $[a, b, c, d]$ 、 $[a, c, b, d]$ 和 $[a, b, d, c]$

第一種： $[a, b, c, d]$



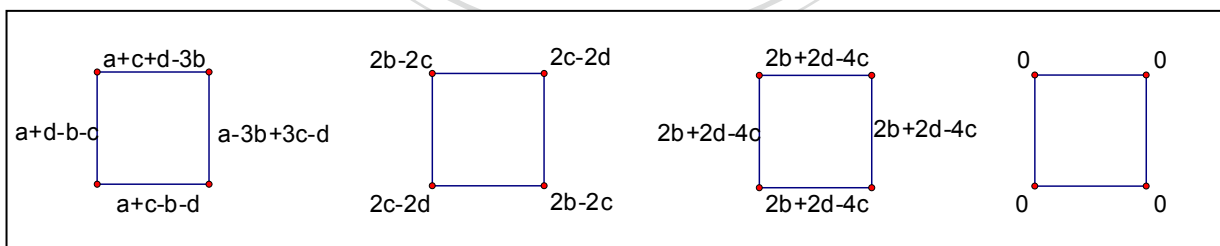
因為  $a + c - 2b$  與  $b + d - 2c$  的值不確定，所以討論下列 9 種情形：

(一) 若  $a + c > 2b$  且  $b + d > 2c$



繼續討論  $a + c + d - 3b$  與  $(a - d) - 3(b - c)$  的正、負號情形：

① 如果  $a + c + d > 3b$  且  $(a - d) > 3(b - c)$



所以如果  $a + c + d > 3b$  且  $(a - d) > 3(b - c)$ ， $l[a, b, c, d] = 6$

② 如果  $a + c + d > 3b$  且  $(a - d) = 3(b - c)$

因為  $a = 3b - 3c + d$

則  $a + c + d - 3b = 3b - 3c + d + c + d - 3b = -2c + 2d < 0$

所以此情形不會發生

③如果  $a+c+d > 3b$  且  $(a-d) < 3(b-c)$

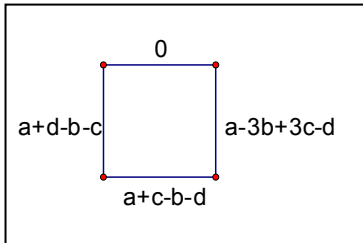
$$a+c+d > 3b$$

$$+) \quad 3b-3c > a-d$$

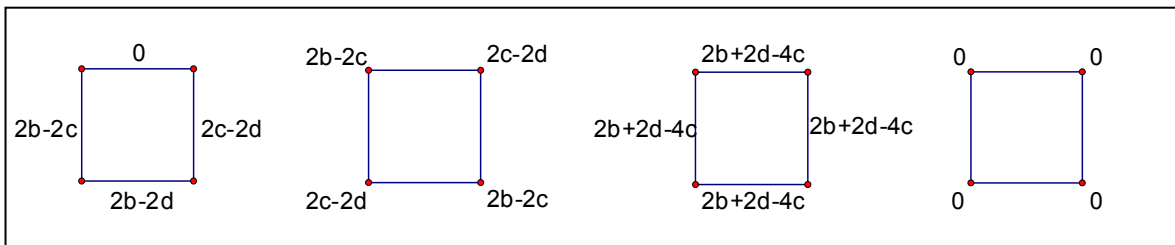
$$2d > 2c \quad (\text{矛盾})$$

所以此情形也不會發生

④如果  $a+c+d = 3b$  且  $(a-d) > 3(b-c)$



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所以如果  $a+c+d = 3b$  且  $(a-d) > 3(b-c)$  ,  $l[a,b,c,d] = 6$

⑤如果  $a+c+d = 3b$  且  $(a-d) = 3(b-c)$

因為  $a = 3b - c - d$

則  $a - d - 3b + 3c = 3b - c - d - d - 3b + 3c = 2c - 2d > 0$

所以此情形也不會發生

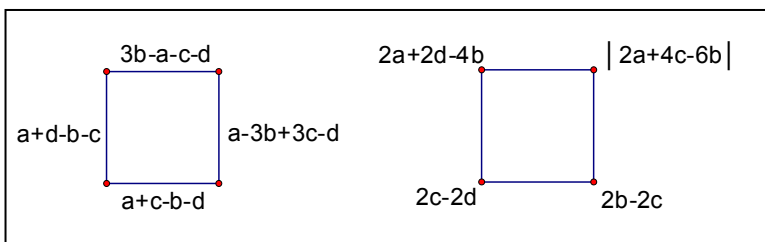
⑥如果  $a+c+d = 3b$  且  $(a-d) < 3(b-c)$

因為  $a = 3b - c - d$

則  $a - d - 3b + 3c = 3b - c - d - d - 3b + 3c = 2c - 2d > 0$

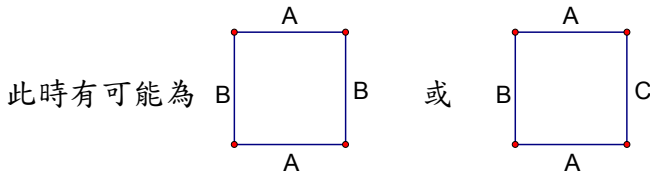
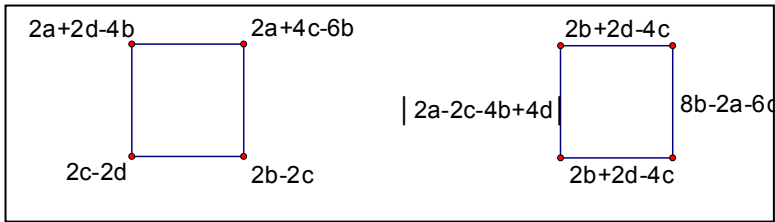
所以此情形也不會發生

⑦如果  $a+c+d < 3b$  且  $(a-d) > 3(b-c)$

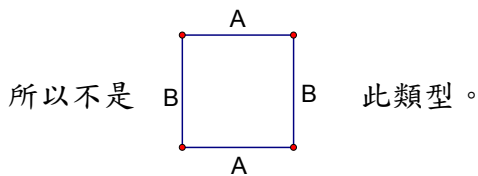


繼續討論  $2a + 4c - 6b$  的情形：

❶ 如果  $2a + 4c > 6b$



$$\text{if } |2(a-c) - 4(b-d)| = 8b - 2a - 6c \Rightarrow \begin{cases} 2a - 2c - 4b + 4d = 8b - 2a - 6c \Rightarrow a + c + d = 3b (\text{矛盾}) \\ -2a + 2c + 4b - 4d = 8b - 2a - 6c \Rightarrow b + d = 2c (\text{矛盾}) \end{cases}$$

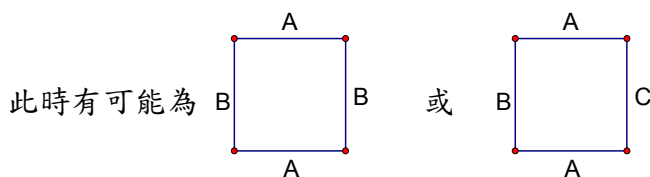
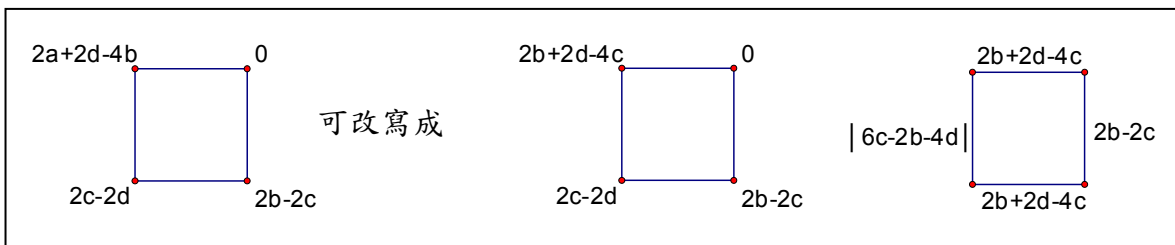


$$\text{if } |2(a-c) - 4(b-d)| + 8b - 2a - 6c = 2(2b + 2d - 4c)$$

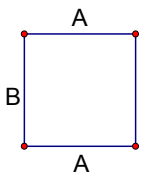
$$\Rightarrow \begin{cases} 2a - 2c - 4b + 4d + 8b - 2a - 6c = 4b + 4d - 8c \Rightarrow 0 = 0 \\ 0 + 8b - 2a - 6c = 4b + 4d - 8c \Rightarrow 2(a-c) = 4(b-d) \\ -2a + 2c + 4b - 4d + 8b - 2a - 6c = 4b + 4d - 8c \Rightarrow 2(a-c) = 4(b-d) (\text{矛盾}) \end{cases}$$

$$\text{所以 } \begin{cases} \text{if } a - c \geq 2(b - d) \Rightarrow l[a, b, c, d] = 7 \\ \text{if } a - c < 2(b - d) \Rightarrow l[a, b, c, d] = 9 \end{cases}$$

❷ 如果  $2a + 4c = 6b$



$$\text{if } |6c - 2b - 4d| = 2b - 2c \Rightarrow \begin{cases} 6c - 2b - 4d = 2b - 2c \Rightarrow b + d = 2c (\text{矛盾}) \\ -6c + 2b + 4d = 2b - 2c \Rightarrow c = d (\text{矛盾}) \end{cases}$$

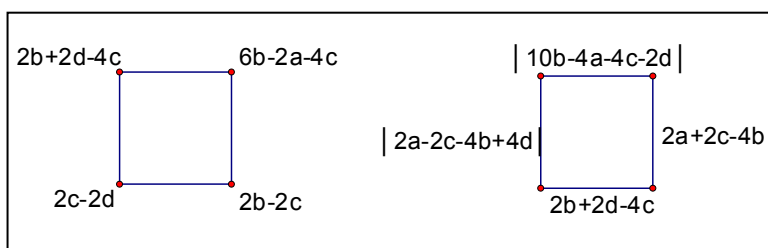
所以不是  此類型。

$$\text{if } |6c - 2b - 4d| + 2b - 2c = 2(2b + 2d - 4c)$$

$$\Rightarrow \begin{cases} 6c - 2b - 4d + 2b - 2c = 4b + 4d - 8c \Rightarrow 6c = 2b + 4d (\text{矛盾}) \\ 0 + 2b - 2c = 4b + 4d - 8c \Rightarrow 6c = 2b + 4d \\ -6c + 2b + 4d + 2b - 2c = 4b + 4d - 8c \Rightarrow 0 = 0 \end{cases}$$

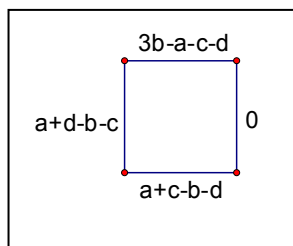
所以  $\begin{cases} \text{if } b + 2d \geq 3c \Rightarrow l[a, b, c, d] = 7 \\ \text{if } b + 2d < 3c \Rightarrow l[a, b, c, d] = 9 \end{cases}$

③ 如果  $2a + 4c < 6b$

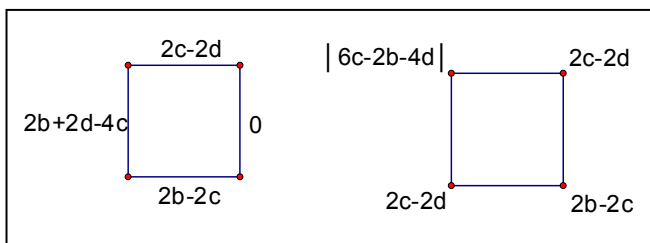


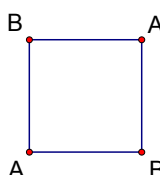
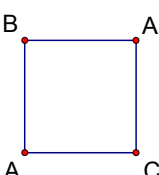
此情形的迪菲方塊(DIFFY BOX)長度可能為七或九，甚至更多。

④ 如果  $a + c + d < 3b$  且  $(a - d) = 3(b - c)$

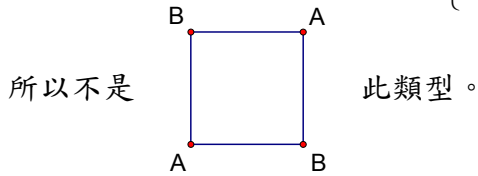


可改寫成



此時有可能為  或 

$$\text{if } |6c - 2b - 4d| = 2b - 2c \Rightarrow \begin{cases} 6c - 2b - 4d = 2b - 2c \Rightarrow b + d = 2c (\text{矛盾}) \\ -6c + 2b + 4d = 2b - 2c \Rightarrow c = d (\text{矛盾}) \end{cases}$$

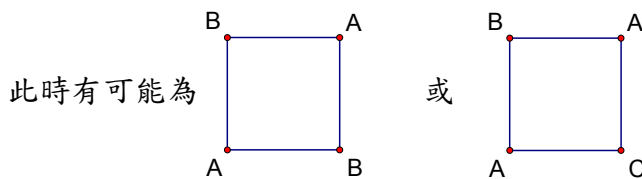
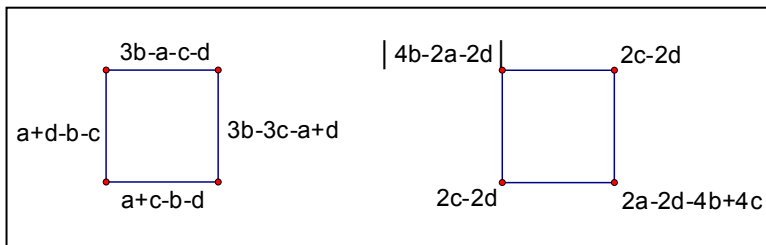


$$\text{if } |6c - 2b - 4d| + 2b - 2c = 2(2c - 2d)$$

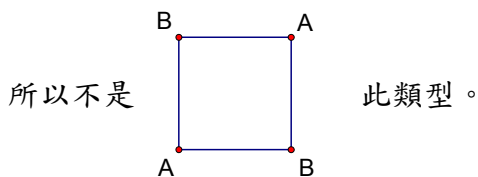
$$\Rightarrow \begin{cases} 6c - 2b - 4d + 2b - 2c = 4c - 4d \Rightarrow 0 = 0 \\ 0 + 2b - 2c = 4c - 4d \Rightarrow 6c = 2b + 4d \\ -6c + 2b + 4d + 2b - 2c = 4c - 4d \Rightarrow 6c = 2b + 4d (\text{矛盾}) \end{cases}$$

$$\text{所以 } \begin{cases} \text{if } b + 2d \leq 3c \Rightarrow l[a, b, c, d] = 6 \\ \text{if } b + 2d > 3c \Rightarrow l[a, b, c, d] = 8 \end{cases}$$

⑨ 如果  $a + c + d < 3b$  且  $(a - d) < 3(b - c)$



$$\text{if } |4b - 2a - 2d| = 2(a - d) - 4(b - c) \Rightarrow \begin{cases} 4b - 2a - 2d = 2a - 2d - 4b + 4c \Rightarrow a + c = 2b (\text{矛盾}) \\ -4b + 2a + 2d = 2a - 2d - 4b + 4c \Rightarrow c = d (\text{矛盾}) \end{cases}$$



$$\text{if } |4b - 2a - 2d| + 2(a - d) - 4(b - c) = 2(2c - 2d)$$

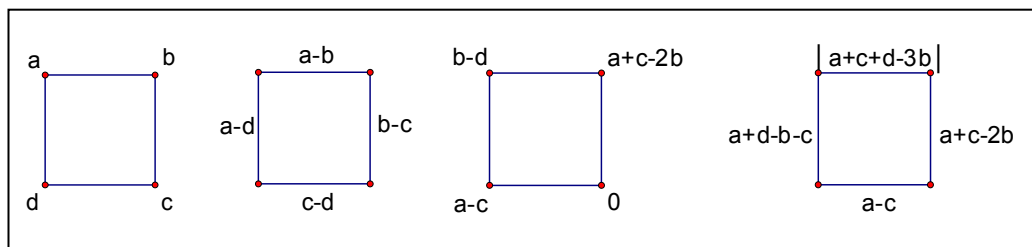
$$\Rightarrow \begin{cases} 4b - 2a - 2a + 2a - 2a - 4b + 4c = 4c - 4d \Rightarrow 0 = 0 \\ 0 + 2a - 2d - 4b + 4c = 4c - 4d \Rightarrow 4b = 2a + 2d \\ -4b + 2a + 2d + 2a - 2d - 4b + 4d = 4c - 4d \Rightarrow 4b = 2a + 2d (\text{矛盾}) \end{cases}$$

$$\text{所以 } \begin{cases} \text{if } a + d \leq 2b \Rightarrow l[a, b, c, d] = 6 \\ \text{if } a + d > 2b \Rightarrow l[a, b, c, d] = 8 \end{cases}$$

**結論：**若  $a+c > 2b$  且  $b+d > 2c$

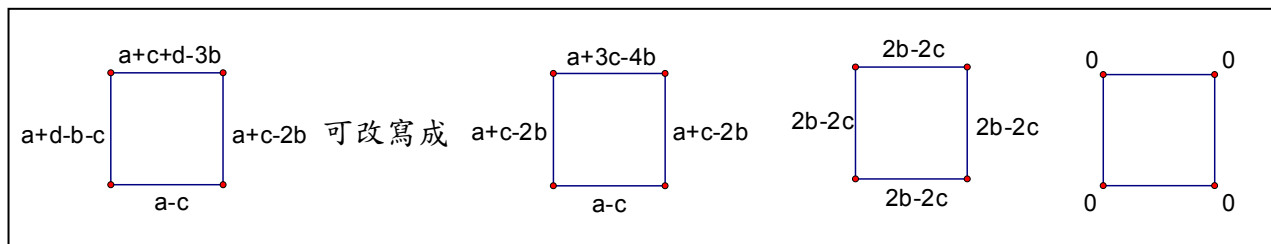
$$\left\{ \begin{array}{l} a+c+d \geq 3b \text{ 且 } (a-d) > 3(b-c) \Rightarrow l[a,b,c,d] = 6 \\ a+c+d < 3b \text{ 且 } (a-d) > 3(b-c) \left\{ \begin{array}{l} 3b > a+2c \Rightarrow l[a,b,c,d] \geq 7 \\ 3b = a+2c \left\{ \begin{array}{l} b+2d \geq 3c \Rightarrow l[a,b,c,d] = 7 \\ b+2d < 3c \Rightarrow l[a,b,c,d] = 9 \end{array} \right. \\ 3b < a+2c \left\{ \begin{array}{l} a-c \geq 2(b-d) \Rightarrow l[a,b,c,d] = 7 \\ a-c < 2(b-d) \Rightarrow l[a,b,c,d] = 9 \end{array} \right. \end{array} \right. \\ a-d = 3(b-c) \left\{ \begin{array}{l} b+2d \leq 3c \Rightarrow l[a,b,c,d] = 6 \\ b+2d > 3c \Rightarrow l[a,b,c,d] = 8 \end{array} \right. \\ a+c+d < 3b \text{ 且 } (a-d) < 3(b-c) \left\{ \begin{array}{l} a+d \leq 2b \Rightarrow l[a,b,c,d] = 6 \\ a+d > 2b \Rightarrow l[a,b,c,d] = 8 \end{array} \right. \end{array} \right.$$

(二) 若  $a+c > 2b$  且  $b+d = 2c$



繼續討論  $a+c+d-3b$  的正、負號情形：

① 如果  $a+c+d > 3b$



所以  $a+c+d > 3b$  ,  $l[a,b,c,d] = 5$

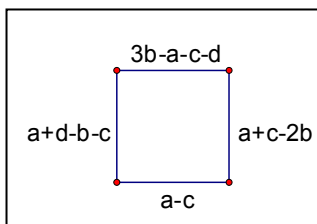
② 如果  $a+c+d = 3b$

因為  $b+d = 2c$

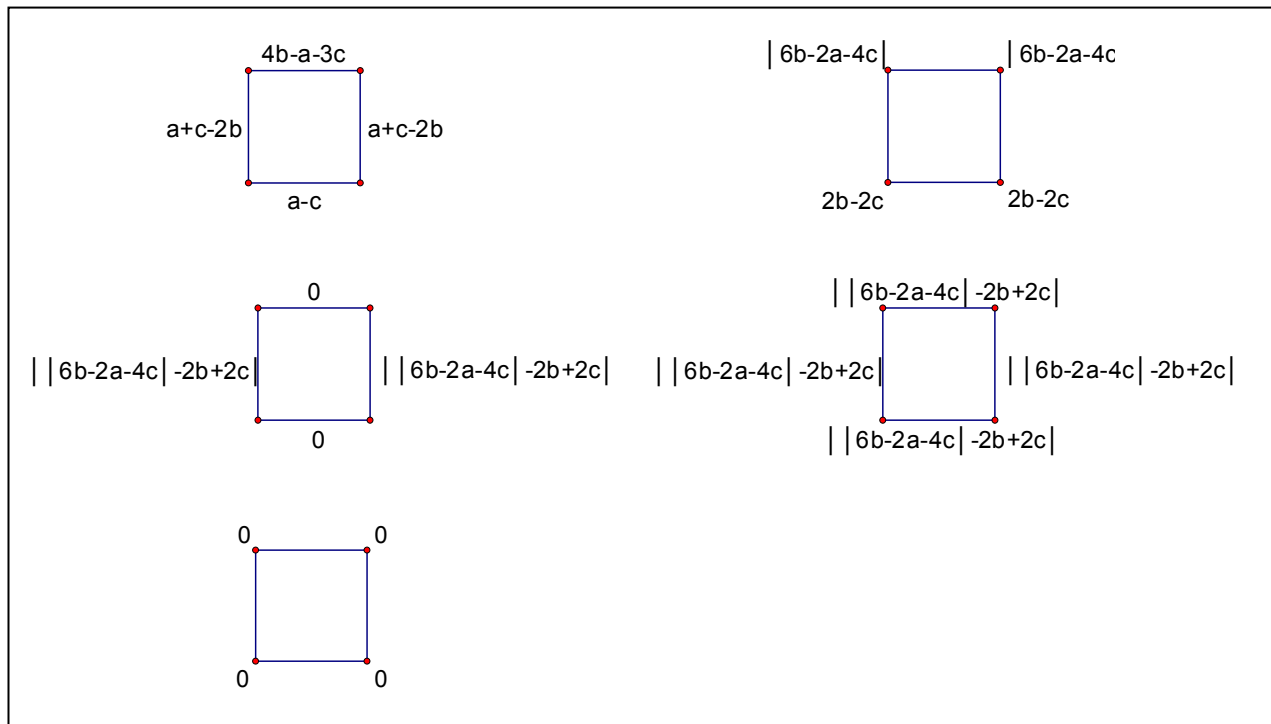
則  $2c - b = 3b - a - c \Rightarrow a + c = 2b$  (\*)

所以此情形不會發生

③ 如果  $a+c+d < 3b$



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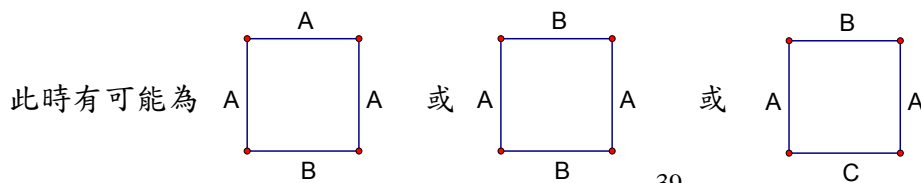
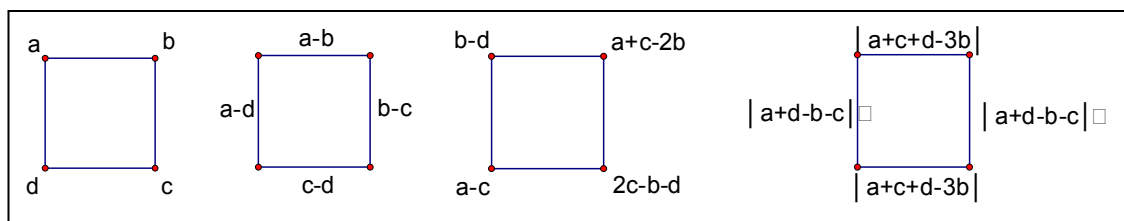


所以  $a+c+d < 3b$  ,  $l[a,b,c,d] = 7$

**結論：** 若  $a+c > 2b$  且  $b+d = 2c$

$\left\{ \begin{array}{l} \text{if } a+c+d > 3b \Rightarrow l[a,b,c,d] = 5 \\ \text{if } a+c+d < 3b \Rightarrow l[a,b,c,d] = 7 \end{array} \right.$

(三) 若  $a+c > 2b$  且  $b+d < 2c$



所以檢查下列式子：

$$\begin{cases} |a+c+d-3b| = |a+d-b-c| \neq |a+b+d-3c| \\ |a+c+d-3b| = |a+b+d-3c| \neq |a+d-b-c| \\ |a+c+d-3b| + |a+b+d-3c| = 2|a+d-b-c| \\ |a+c+d-3b| + |a+b+d-3c| \neq 2|a+d-b-c| \end{cases}$$

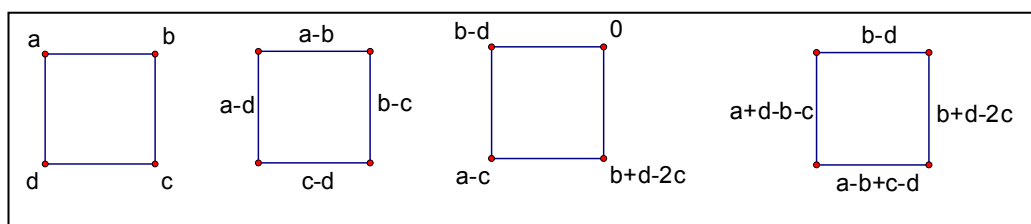
其中  $|a+c+d-3b| = |a+b+d-3c| \neq |a+d-b-c|$  此種情形不會發生，因為

$$\begin{cases} \text{if } a+c+d-3b = a+b+d-3c \Rightarrow b=c (\text{矛盾}) \\ \text{if } a+c+d-3b = -a-b-d+3c \Rightarrow a+d = b+c (\text{矛盾}) \end{cases}$$

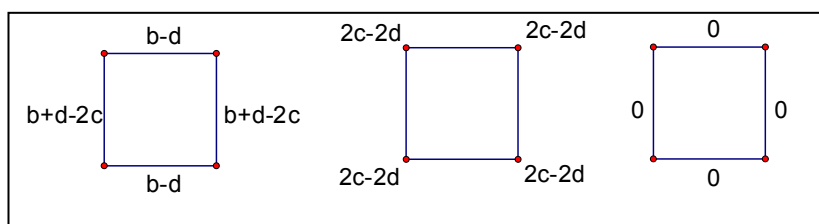
**結論：** 若  $a+c > 2b$  且  $b+d < 2c$

$$\begin{cases} \text{if } |a+c+d-3b| = |a+d-b-c| \neq |a+b+d-3c| \Rightarrow l[a,b,c,d] = 7 \\ \text{if } |a+c+d-3b| + |a+b+d-3c| = 2|a+d-b-c| \Rightarrow l[a,b,c,d] = 5 \\ \text{if } |a+c+d-3b| + |a+b+d-3c| \neq 2|a+d-b-c| \Rightarrow l[a,b,c,d] = 7 \end{cases}$$

(四) 若  $a+c = 2b$  且  $b+d > 2c$



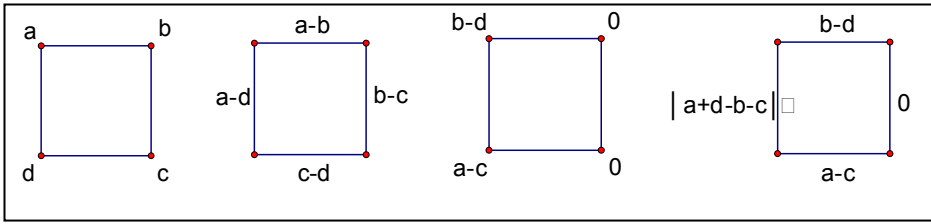
可改寫成



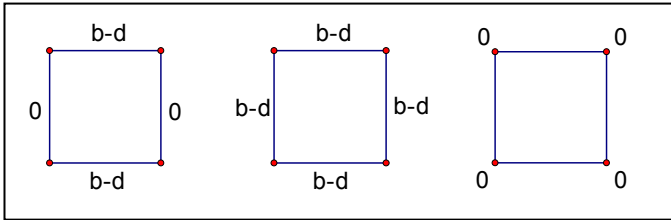
**結論：** 若  $a+c = 2b$  且  $b+d > 2c \Rightarrow l[a,b,c,d] = 5$



(五) 若  $a+c=2b$  且  $b+d=2c$

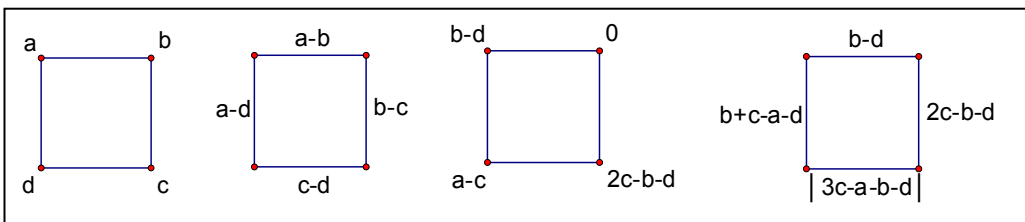


可改寫成



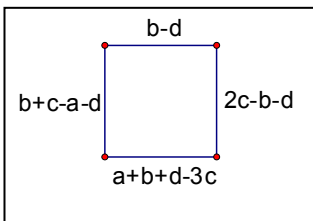
**結論：** 若  $a+c=2b$  且  $b+d=2c \Rightarrow l[a,b,c,d]=5$

(六) 若  $a+c=2b$  且  $b+d < 2c$

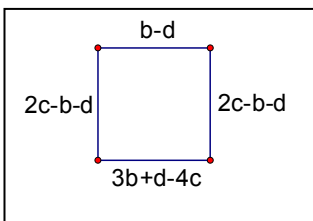


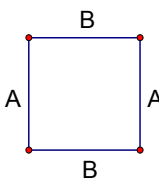
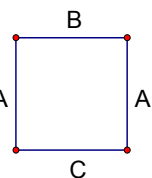
繼續討論  $a+b+d-3c$  的正、負號情形：

① 如果  $a+b+d > 3c$

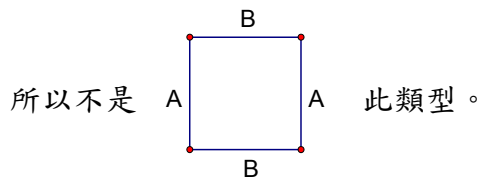


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此時有可能為  或 

如果  $3b+d-4c=b-d \Rightarrow b+d=2c$  (矛盾)

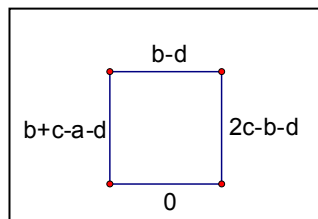


又如果  $3b + d - 4c + b - d = 2(2c - b - d) \Rightarrow 3b + d - 4c = 0$  (矛盾)

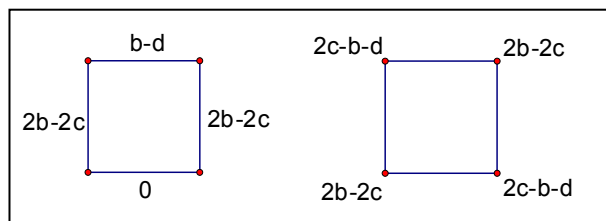
所以  $(3b + d - 4c) + (b - d) \neq 2(2c - b - d)$

因此  $l[a, b, c, d] = 7$ 。

② 如果  $a + b + d = 3c$



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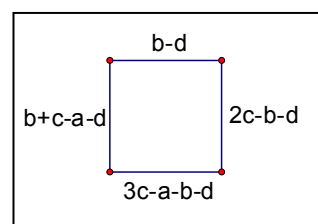


因為  $a + b + d = 3c$  且  $a + c = 2b \Rightarrow 3b + d = 4c$

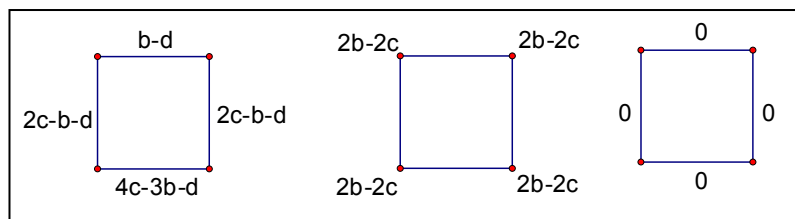
也就是  $2c - b - d = 2b - 2c$

所以  $l[a, b, c, d] = 5$ 。

③ 如果  $a + c + d < 3b$



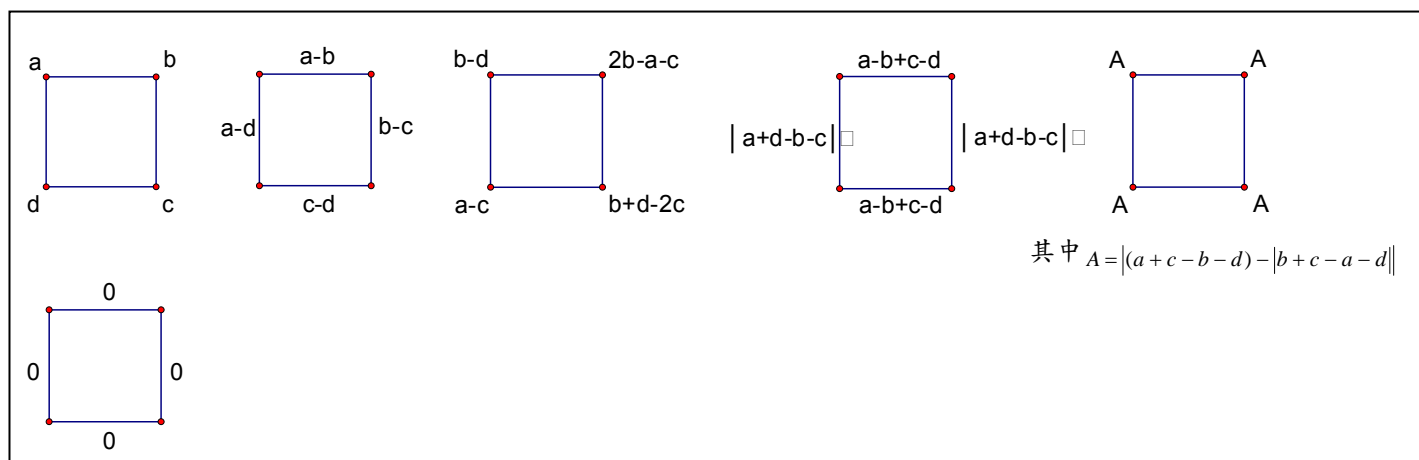
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**結論：** 若  $a + c = 2b$  且  $b + d < 2c$

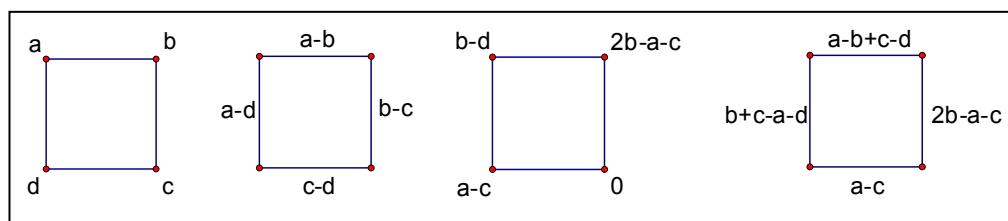
$$\begin{cases} \text{if } a+b+d \leq 3c \Rightarrow l[a,b,c,d]=5 \\ \text{if } a+b+d > 3c \Rightarrow l[a,b,c,d]=7 \end{cases}$$

(七) 若  $a+c < 2b$  且  $b+d > 2c$

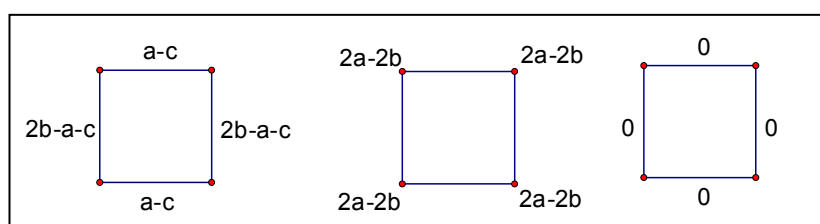


**結論：** 若  $a+c < 2b$  且  $b+d > 2c \Rightarrow l[a,b,c,d]=5$

(八) 若  $a+c < 2b$  且  $b+d = 2c$

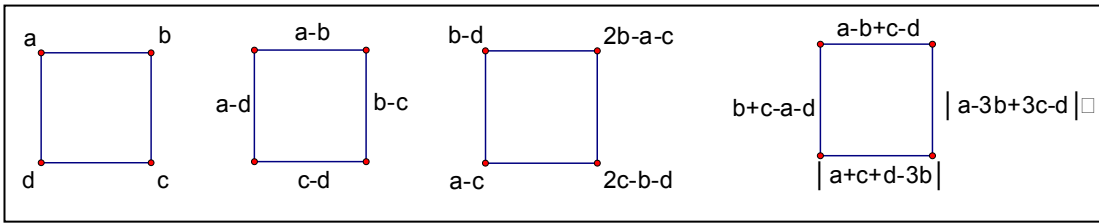


可改寫成



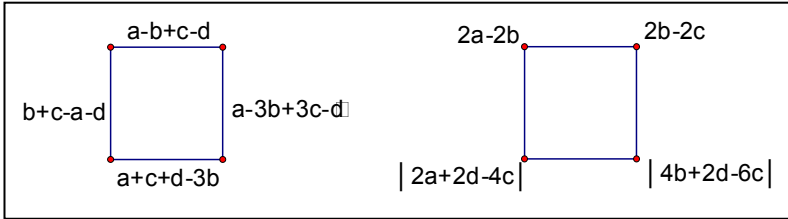
**結論：** 若  $a+c < 2b$  且  $b+d = 2c \Rightarrow l[a,b,c,d]=5$

(九) 若  $a+c < 2b$  且  $b+d < 2c$



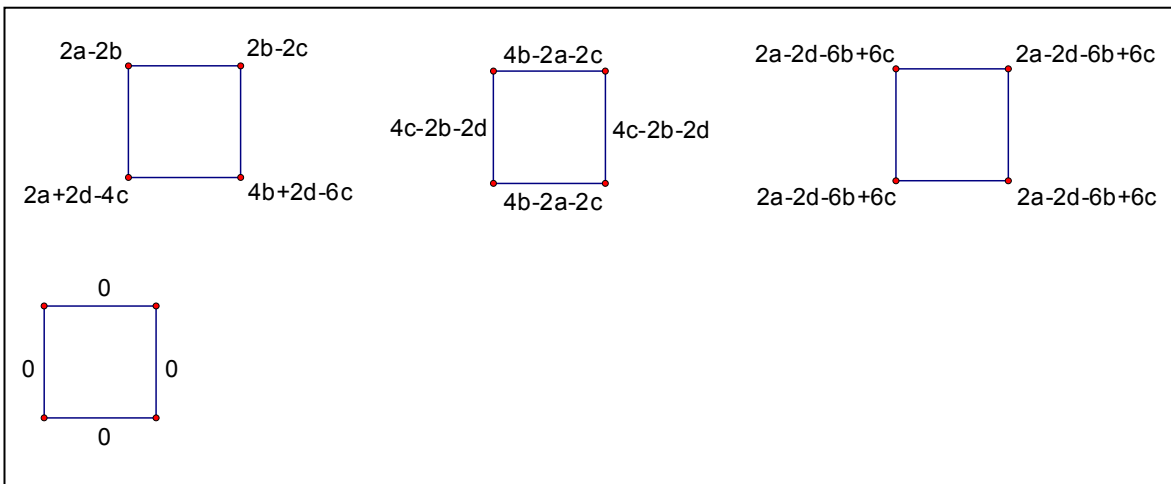
繼續討論  $a+b+d-3c$  與  $(a-d)-3(b-c)$  的正、負號情形：

① 如果  $a+b+d > 3c$  且  $(a-d) > 3(b-c)$



繼續討論  $2a+2d-4c$  與  $4b+2d-6c$  的正、負號情形：

① 如果  $2a+2d > 4c$  且  $4b+2d > 6c$



② 如果  $2a+2d > 4c$  且  $4b+2d = 6c$

因為  $2d = 6c - 4b$

則  $2a+2d-4c = 2a+6c-4b-4c = 2a+2c-4b < 0$  (矛盾)

所以此情形不會發生

③ 如果  $2a+2d > 4c$  且  $4b+2d < 6c$

$$2a+2d > 4c$$

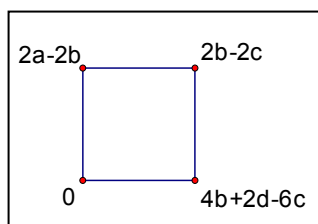
$$+ ) \quad 6c > 4b+2d$$

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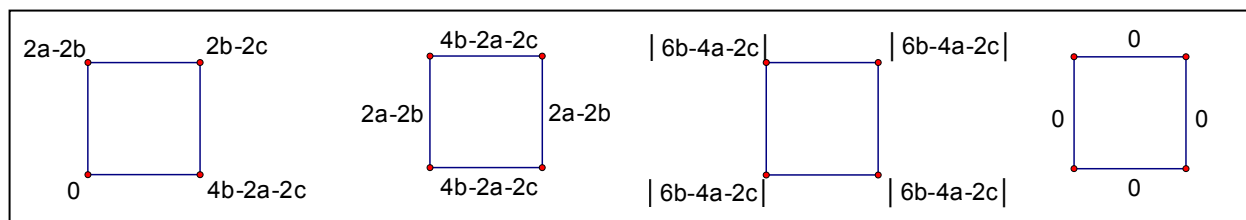

$$a+c > 2b \quad (\text{矛盾})$$

所以此情形也不會發生

④ 如果  $2a + 2d = 4c$  且  $4b + 2d > 6c$



可改寫成



⑤ 如果  $2a + 2d = 4c$  且  $4b + 2d = 6c$

因為  $2d = 4c - 2a$

則  $4b + 2d - 6c = 4b + 4c - 2a - 6c = 4b - 2a - 2c > 0$  (矛盾)

所以此情形也不會發生

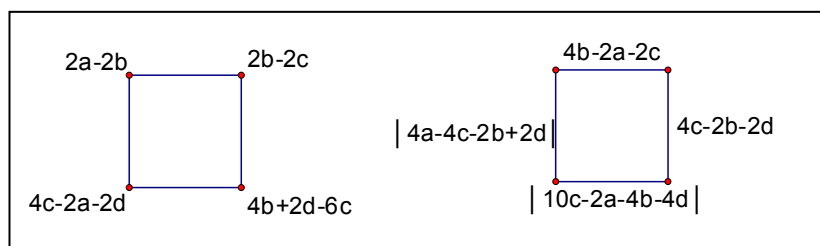
⑥ 如果  $2a + 2d = 4c$  且  $4b + 2d < 6c$

因為  $2d = 4c - 2a$

則  $4b + 2d - 6c = 4b + 4c - 2a - 6c = 4b - 2a - 2c > 0$  (矛盾)

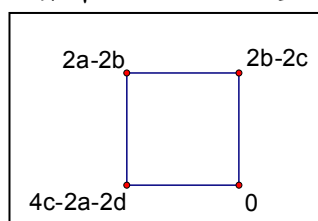
所以此情形也不會發生

⑦ 如果  $2a + 2d < 4c$  且  $4b + 2d > 6c$

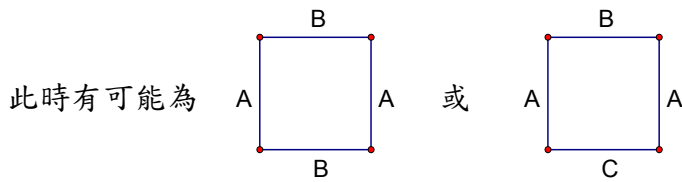
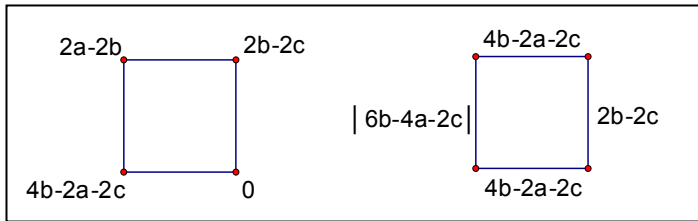


此情形的迪菲方塊(DIFFY BOX)長度可能為八或九，甚至更多。

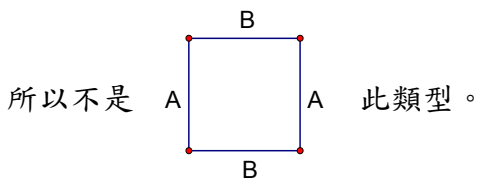
⑧ 如果  $2a + 2d < 4c$  且  $4b + 2d = 6c$



可改寫成



$$\text{if } |6b - 4a - 2c| = 2b - 2c \Rightarrow \begin{cases} 6b - 4a - 2c = 2b - 2c \Rightarrow a = b (\text{矛盾}) \\ -6b + 4a + 2c = 2b - 2c \Rightarrow a + c = 2b (\text{矛盾}) \end{cases}$$

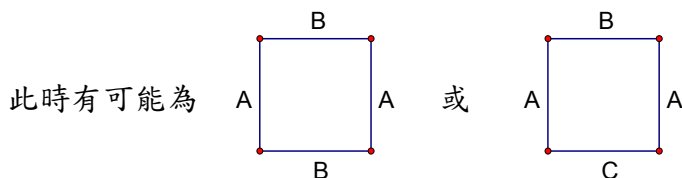
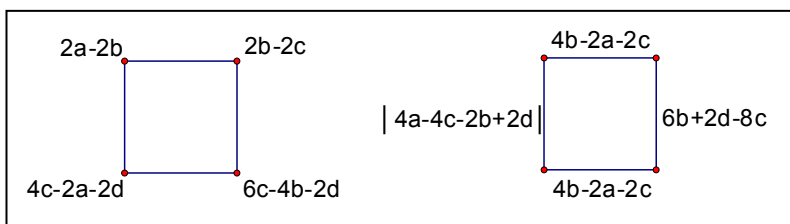


$$\text{if } |6b - 4a - 2c| + 2b - 2c = 2(4b - 2a - 2c)$$

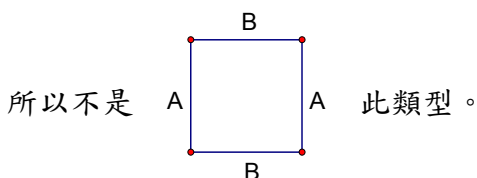
$$\Rightarrow \begin{cases} 6b - 4a - 2c + 2b - 2c = 8b - 4a - 4c \Rightarrow 0 = 0 \\ 0 + 2b - 2c = 8b - 4a - 4c \Rightarrow 6b = 4a + 2c \\ -6b + 4a + 2c + 2b - 2c = 8b - 4a - 4c \Rightarrow 6b = 4a + 2c (\text{矛盾}) \end{cases}$$

$$\text{所以 } \begin{cases} \text{if } 3b \geq 2a + c \Rightarrow l[a, b, c, d] = 7 \\ \text{if } 3b < 2a + c \Rightarrow l[a, b, c, d] = 9 \end{cases}$$

⑨ 如果  $2a + 2d < 4c$  且  $4b + 2d < 6c$



$$\text{if } |4(a - c) - 2(b - d)| = 6b + 2d - 8c \Rightarrow \begin{cases} 4a - 4c - 2b + 2d = 6b + 2d - 8c \Rightarrow a + c = 2b (\text{矛盾}) \\ -4a + 4c + 2b - 2d = 6b + 2d - 8c \Rightarrow a + b + d = 3c (\text{矛盾}) \end{cases}$$

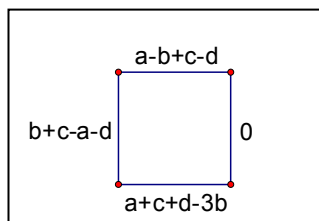


$$\text{if } |4(a-c) - 2(b-d)| + 6b + 2d - 8c = 2(4b - 2a - 2c)$$

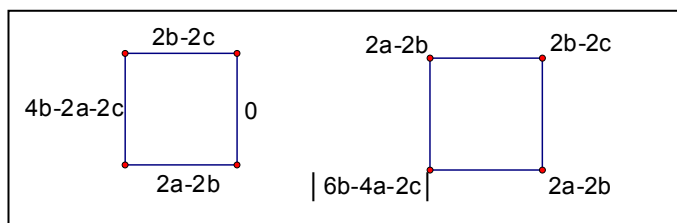
$$\Rightarrow \begin{cases} 4a - 4c - 2b + 2d + 6b + 2d - 8c = 8b - 4a - 4c \Rightarrow 4(a-c) = 2(b-d) \text{ (矛盾)} \\ 0 + 6b + 2d - 8c = 8b - 4a - 4c \Rightarrow 4(a-c) = 2(b-d) \\ -4a + 4c + 2b - 2d + 6b + 2d - 8c = 8b - 4a - 4c \Rightarrow 0 = 0 \end{cases}$$

$$\text{所以 } \begin{cases} \text{if } 2(a-c) \leq b-d \Rightarrow l[a,b,c,d] = 7 \\ \text{if } 2(a-c) > b-d \Rightarrow l[a,b,c,d] = 9 \end{cases}$$

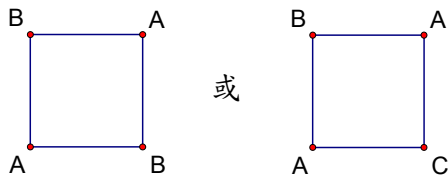
② 如果  $a+b+d > 3c$  且  $(a-d) = 3(b-c)$



可改寫成



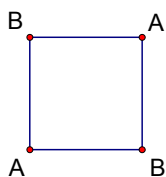
此時有可能為



或

$$\text{if } |6b - 4a - 2c| = 2b - 2c \Rightarrow \begin{cases} 6b - 4a - 2c = 2b - 2c \Rightarrow a = b \text{ (矛盾)} \\ -6b + 4a + 2c = 2b - 2c \Rightarrow a + c = 2b \text{ (矛盾)} \end{cases}$$

所以不是



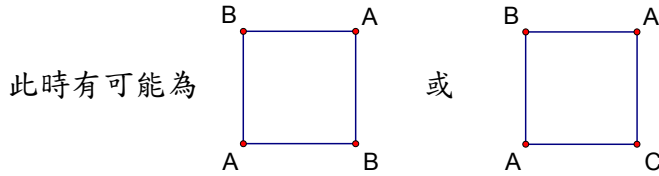
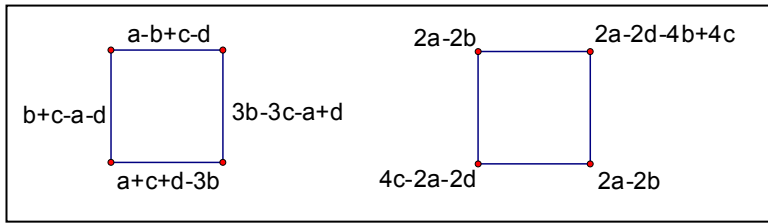
此類型。

$$\text{if } |6b - 4a - 2c| + 2b - 2c = 2(2a - 2b)$$

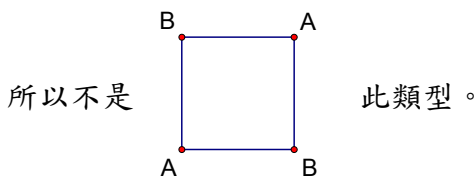
$$\Rightarrow \begin{cases} 6b - 4a - 2c + 2b - 2c = 4a - 4b \Rightarrow 6b = 4a + 2c \text{ (矛盾)} \\ 0 + 2b - 2c = 4a - 4b \Rightarrow 6b = 4a + 2c \\ -6b + 4a + 2c + 2b - 2c = 4a - 4b \Rightarrow 0 = 0 \end{cases}$$

$$\text{所以 } \begin{cases} \text{if } 2a + c \geq 3b \Rightarrow l[a,b,c,d] = 6 \\ \text{if } 2a + c < 3b \Rightarrow l[a,b,c,d] = 8 \end{cases}$$

③如果  $a+b+d > 3c$  且  $(a-d) < 3(b-c)$



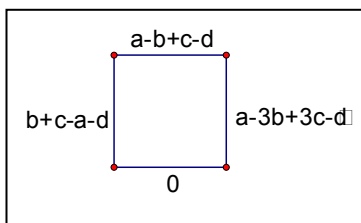
if  $2(a-d) - 4(b-c) = 4c - 2a - 2d \Rightarrow a = b$  (矛盾)



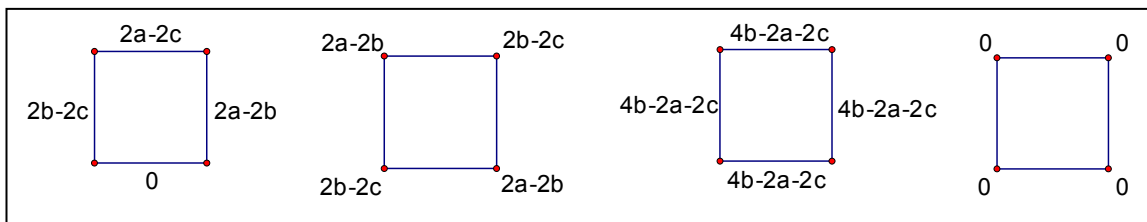
if  $2(a-d) - 4(b-c) + 4c - 2a - 2d = 2(2a - 2b) \Rightarrow a + d = 2c$

所以  $\begin{cases} \text{if } a + d = 2c \Rightarrow l[a, b, c, d] = 6 \\ \text{if } a + d \neq 2c \Rightarrow l[a, b, c, d] = 8 \end{cases}$

④如果  $a+b+d = 3c$  且  $(a-d) > 3(b-c)$



可改寫成



⑤如果  $a+b+d = 3c$  且  $(a-d) = 3(b-c)$

因為  $d = 3c - a - b$

則  $a - d - 3b + 3c = a - 3c + a + b - 3b + 3c = 2a - 2b > 0$  (矛盾)

所以此情形也不會發生



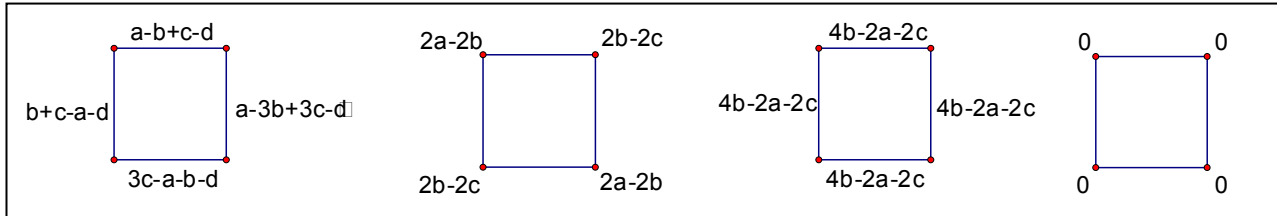
⑥ 如果  $a+b+d = 3c$  且  $(a-d) < 3(b-c)$

因為  $d = 3c - a - b$

則  $a - d - 3b + 3c = a - 3c + a + b - 3b + 3c = 2a - 2b > 0$  (矛盾)

所以此情形也不會發生

⑦ 如果  $a+b+d < 3c$  且  $(a-d) > 3(b-c)$



⑧ 如果  $a+b+d < 3c$  且  $(a-d) = 3(b-c)$

因為  $d = a - 3b + 3c$

則  $a + b + d - 3c = a + b + a - 3b + 3c - 3c = 2a - 2b > 0$  (矛盾)

所以此情形也不會發生

⑨ 如果  $a+b+d < 3c$  且  $(a-d) < 3(b-c)$

$$a + b + d < 3c$$

$$+) \quad a - d < 3b - 3c$$

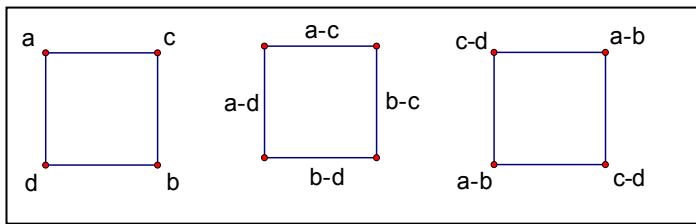
$$2a < 2b \quad (\text{矛盾})$$

所以此情形也不會發生

**結論：** 若  $a+c < 2b$  且  $b+d < 2c$

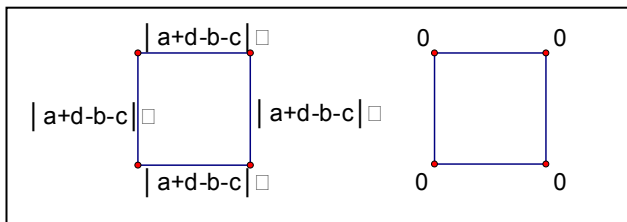
$$\left\{ \begin{array}{l} a+b+d > 3c \text{ 且 } (a-d) > 3(b-c) \\ a+b+d > 3c \text{ 且 } (a-d) = 3(b-c) \\ a+b+d > 3c \text{ 且 } (a-d) < 3(b-c) \\ a+b+d \leq 3c \end{array} \right. \left\{ \begin{array}{l} a+d \geq 2c \text{ 且 } 2b+d > 3c \Rightarrow l[a,b,c,d] = 7 \\ a+d < 2c \text{ 且 } 2b+d > 3c \Rightarrow l[a,b,c,d] \geq 8 \\ a+d < 2c \text{ 且 } 2b+d = 3c \left\{ \begin{array}{l} 3b \geq 2a+c \Rightarrow l[a,b,c,d] = 7 \\ 3b < 2a+c \Rightarrow l[a,b,c,d] = 9 \end{array} \right. \\ a+d < 2c \text{ 且 } 2b+d < 3c \left\{ \begin{array}{l} 2(a-c) \leq (b-d) \Rightarrow l[a,b,c,d] = 7 \\ 2(a-c) > (b-d) \Rightarrow l[a,b,c,d] = 9 \end{array} \right. \\ 3b \leq 2a+c \Rightarrow l[a,b,c,d] = 6 \\ 3b > 2a+c \Rightarrow l[a,b,c,d] = 8 \\ a+d = 2c \Rightarrow l[a,b,c,d] = 6 \\ a+d \neq 2c \Rightarrow l[a,b,c,d] = 8 \end{array} \right.$$

第二種：  $[a, c, b, d]$



若  $a - b = c - d$  即  $a + d = b + c \Rightarrow l[a, c, b, d] = 3$

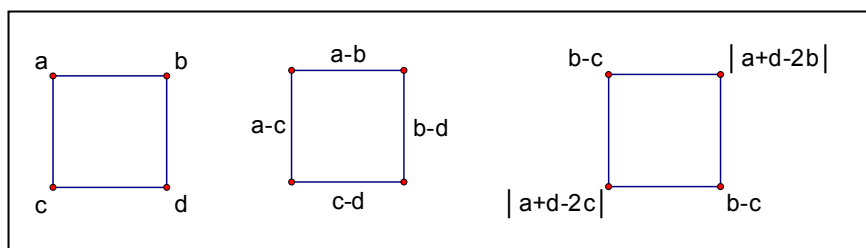
若  $a - b \neq c - d$ ，迪菲方塊(DIFFY BOX)可繼續運算如下：



**結論：**

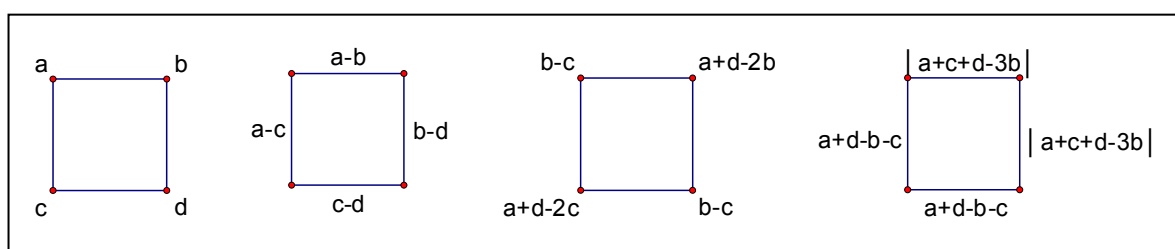
$$\begin{cases} \text{if} & a + d = b + c \Rightarrow l[a, c, b, d] = 3 \\ \text{if} & a + d \neq b + c \Rightarrow l[a, c, b, d] = 4 \end{cases}$$

第三種：  $[a, b, d, c]$



繼續討論  $a+d-2b$  與  $a+d-2c$  的正、負號情形：

(一) 如果  $a+d > 2b > 2c$

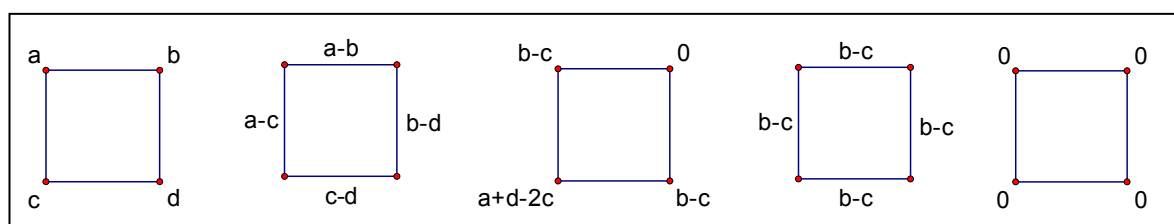


如果  $|a+c+d-3b| = a+d-b-c \Rightarrow \begin{cases} a+c+d-3b = a+d-b-c \Rightarrow b=c (\text{矛盾}) \\ -a-c-d+3b = a+d-b-c \Rightarrow a+d=2b (\text{矛盾}) \end{cases}$

所以  $|a+c+d-3b| \neq a+d-b-c$

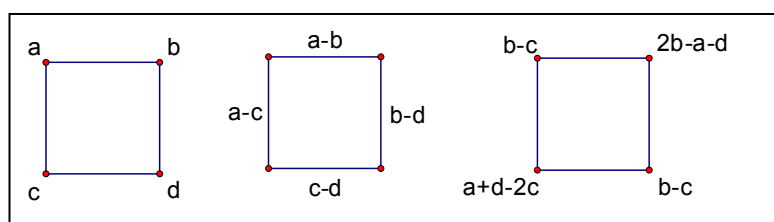
因此如果  $a+d > 2b > 2c$  ,  $l[a, b, d, c] = 6$

(二) 如果  $a+d = 2b > 2c$



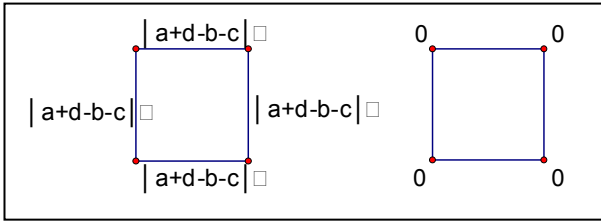
因此如果  $a+d = 2b > 2c$  ,  $l[a, b, d, c] = 4$

(三) 如果  $2b > a+d > 2c$



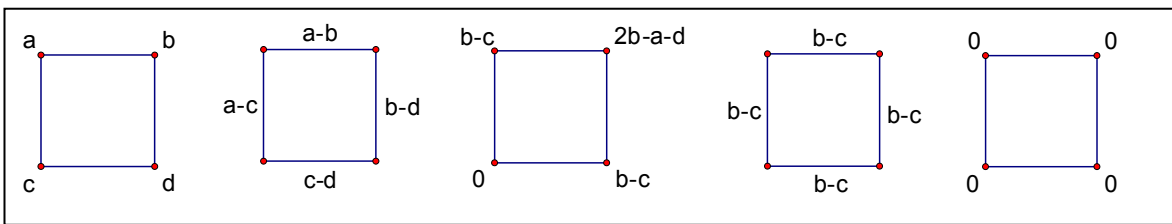
如果  $b - c = 2b - a - c$  即  $a + d = b + c \Rightarrow l[a, b, d, c] = 3$

如果  $b - c \neq 2b - a - c$ ，第三層方塊可繼續運算如下：



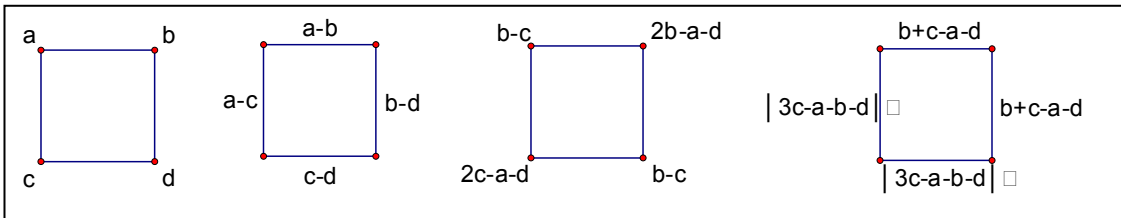
因此如果  $2b > a + d > 2c$ ， $\begin{cases} a + d = b + c \Rightarrow l[a, b, d, c] = 3 \\ a + d \neq b + c \Rightarrow l[a, b, d, c] = 4 \end{cases}$

(四) 如果  $2b > a + d = 2c$



因此如果  $2b > a + d = 2c$ ， $l[a, b, d, c] = 4$

(五) 如果  $2b > 2c > a + d$



如果  $|3c - a - b - d| = b + c - a - d \Rightarrow \begin{cases} 3c - a - b - d = b + c - a - d \Rightarrow b = c (\text{矛盾}) \\ -3c + a + b + d = b + c - a - d \Rightarrow a + d = 2c (\text{矛盾}) \end{cases}$

所以  $|3c - a - b - d| \neq b + c - a - d$

因此如果  $2b > 2c > a + d$ ， $l[a, b, d, c] = 6$

**結論：**

$\begin{cases} a + d > 2b > 2c \Rightarrow l[a, b, d, c] = 6 \\ a + d = 2b > 2c \Rightarrow l[a, b, d, c] = 4 \\ 2b > a + d > 2c \begin{cases} a + d = b + c \Rightarrow l[a, b, d, c] = 3 \\ a + d \neq b + c \Rightarrow l[a, b, d, c] = 4 \end{cases} \\ 2b > a + d = 2c \Rightarrow l[a, b, d, c] = 4 \\ 2b > 2c > a + d \Rightarrow l[a, b, d, c] = 6 \end{cases}$