試科目 Course	線性代數	開課系級 Dept, & Class	應數	日期 Date, Period	月 日 第 節

7. $A=\begin{bmatrix}-10 & 4\\ -25 & 10\end{bmatrix}$, 求

(a) A 的 Jordan 基底 ?

(b) A 的 Jordan 標準型(Canonical Form) ?

8. 設 $A=\begin{bmatrix}2 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 2\end{bmatrix}$, $\mathrm{b}=\begin{bmatrix}6\\ 0\\ -6\end{bmatrix}$

(a) 試求出下三角矩陣 L,上三角矩陣 U 及對角矩陣 D 使得 $A=LDU$.

(b) 求 x 使得 $A\mathrm{x}=\mathrm{b}$.

考試科目 Course	微積分	開課系級 Dept, & Class	應數所	日期 Date, Period	4 月 27 日 第 1 節	試題編號 CourseNo.	

一. Let $h:[0,1]\longrightarrow\mathbb{R}$ be a continuous function, $h(0)=0$ and $\lim_{\delta\to 0^+}\frac{h(s+\delta)-h(s)}{\delta}\le c$ for all $0\le s<1$, where c is a constant. Then $h(s)\le cs$ for all $0\le s\le 1$. (15%)

二. Let f and g be continuous functions on $[a,b]$. Show that the Cauchy-Schwarz inequality holds, i.e.,

$$\int_a^b f(x)g(x)dx \le \left(\int_a^b f(x)^2dx\right)^{1/2}\left(\int_a^b g(x)^2dx\right)^{1/2}.$$ (15%)

三. Let R be a region in $\mathbb{R}^2$ bounded by a peicewise C^1 Jordan curve C. Show that the area of R is

$$\frac{1}{2}\oint_C -y\,dx + x\,dy .$$ (15%)

四. Evaluate $\lim_{n\to\infty}\left(\int_0^1 (3-x^4)^n dx\right)^{\frac{1}{n}}$ if exists. (15%)

五. Let $a_n\geq 0$, $n=1,2,\cdots$. Show that either $\sum_{n=1}^{\infty}a_n$ converges or diverges to $+\infty$. (10%)

六. Evaluate $\iiint_S (x^2+y^2+z^2)\,dx\,dy\,dz$, where $S=\{(x,y,z)\in\mathbb{R}^3 \mid x^2+y^2+z^2\le 1\}$ (10%)

七. Find the distance from a point to a plane in the space. (You should justify your answer). (10%)

八. Using total differential to approximate $(1.001)^5 e^{-0.01}\ln(1.02)$ (10%)

考試科目 Course	機率與統計	開課系級 Dept. & Class	應用數學	日 期 Date, Period	4 月 27 日 第 3 節	試題編號 CourseNo.	

1. If a bank has male and female customers. Their arriving follow Poisson processes with parameters λ_1 and λ_2, respectively. Let X_1 and X_2 denote the waiting time until the first male and female customer arrive after 10 A.M., respectively.
   (a) What is the p.d.f. X_1 and X_2. (5%)
   (b) Prove that the probability that the first customer arrive after 10 A.M. will be a male is equal to $\frac{\lambda_1}{\lambda_1+\lambda_2}$. (10%)

2. A pound contains 100 fishes, of which 30 are carps. If 20 fishes are caught, what are the mean and variance of the number of carps among these 20? What **assumptions** are you making? (15%)

3. If X and Y are independent random variables having identical density functions $f(x)=e^{-x}$, $0<x<\infty$, derive the joint density of $U=X+Y$, $V=X/(X+Y)$. (15%)

4. Suppose that X_1 and X_2 are independent random variables having a common mean μ. Suppose also that $Var(X_1)=\sigma_1^2$ and $Var(X_2)=\sigma_2^2$. The value μ is unknown and it is proposed to estimate μ by a weighted average of X_1 and X_2, *i.e.* $\lambda X_1+(1-\lambda)X_2$. Explain why it is desirable to use this estimator. How to select a λ such that the estimator have the smallest variance? (10%)

5. Let $X_1, X_2, \cdots, X_n$ be a random sample from a uniform distribution on the interval $(0,\ \theta]$.
   (a) Find the method of moments estimator for θ. (5%)
   (b) Find the maximum likelihood estimator for θ. (5%)

6. A certain size bag is designed to hold 25 pounds of potatoes. A farmer fills such bags in the filed. Assume that the weight X of potatoes in a bag is $N(\mu, 9)$. We shall test the null hypothesis H_0: $\mu=25$ against the alternative hypothesis H_1: $\mu<25$. Let X_1, X_2, X_3, X_4 be a random sample of size 4 from this distribution, and let the critical region C for this test be defined by $\bar{x}\le 22.5$, where $\bar{x}$ is the observed value of $\bar{X}$.
   (a) Determine the significance level α of the test. (5%)
   (b) Find the probability of the type II error if in fact $\mu=23$. (5%)
   (c) Give the power function of this test. (5%)

考試科目 Course	機率與統計	開課系級 Dept. & Class		日期 Date, Period	4 月 27 日 第 3 節	試題編號 Course No.	

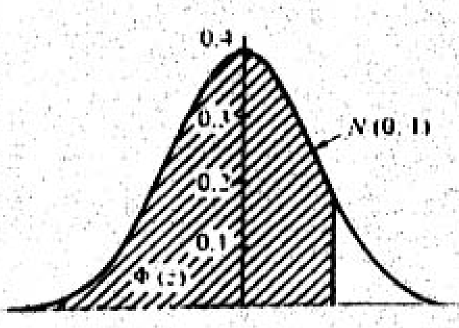
7. Given the five pairs of (x, y) values:

x	0	1	6	3	5
y	4	3	0	2	1

(a) Fit a simple linear regression model to these data. (5%)

(b) Perform an analysis of variance for regression from these data. (10%)

(c) Estimate the expected y value corresponding to $x = 2$, and give a 90% confidence interval. (5%)



The Normal Distribution

$$P(Z \le z) = \Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}\, dw$$

$$[\Phi(-z) = 1 - \Phi(z)]$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

考試科目 Course	作業研究	開課系級 Dept. & Class	應數所	日期 Date, Period	月 日 第 節	試題編號 Course No.

1. Consider the following linear programming problem (LP). (25%)

$$\min z = -x_1 + 7x_2 - 3x_3 \qquad (LP)$$
$$\text{subject to} \quad 2x_1 + x_2 - x_3 \le 4$$
$$4x_1 - 3x_2 \le 2$$
$$-3x_1 + 2x_2 + 2x_3 \ge 1$$
$$x_1,\ x_2,\ x_3 \ge 0$$

(a) Is (LP) feasible? If it is feasible, solve (LP) and find the optimal value z^*.

(b) Formulate a dual problem (D) to (LP). How do you solve (D)? Find the optimal solutions if it is feasible.

(c) Is the optimal value of (D) equal to z^*? Verify your answer.

2. Each day, Taiwan Oil manufactures four types of gasoline: lead-free 95 (95), lead-free 92 (92), leaded premium (lp), and leaded regular (lr). Because of cleaning and resetting of machinery, the time required to produce a batch of gasoline depends on the type of gasoline last produced. Assume that the last gas produced yesterday precedes the first gas produced today. It takes longer to switch between a lead-free gasoline and a leaded gasoline than it does to switch between two lead-free gasolines. The time (in minutes) required to manufacture each day's gasoline requirements are shown in Table 1. You are asked to determine the order in which the gasolines should be produced with minimum switch times each day. For instance, the order 95-92-lr-lp requires 380 minutes in switching. (25%)

(a) Formulate this problem as a Mathematical program;

(b) Describe your solution procedures systematically;

(c) Find the optimal order and the minimum value.

TABLE 1

Last Produced Gasoline	Gas to Be Next Produced			
	95	92	lr	lp
95	--	50	120	140
92	60	--	140	110
lr	90	130	--	60
lp	130	120	80	--

考試科目 Course	作業研究	開課系級 Dept. & Class	應數所	日 期 Date, Period	月 日 第 節	試題編號 Course No.

3. A fast-food restaurant has one drive-in window. An average of 15 cars per hour arrive at the window. It takes an average of 2 minutes to serve a car. Assume that interarrival and service times are exponential. (25%)

   (a) On the average, how many cars are waiting in line?

   (b) On the average, how long does a car spend at the restaurant (from time of arrival to time service is completed)?

   (c) What fraction of the time are more than 3 cars waiting for service (this includes the car (if any) at the window)?

4. Consider the network in Figure 1 as a maximum flow problem from source (SO) to sink (SI). (25%)

   (a) Find the maximum flow from node SO to node SI;

   (b) How do you define a cut in the network? Find a cut whose capacity equals the maximum flow in the network.

   (c) State the max-flow min-cut theorem.

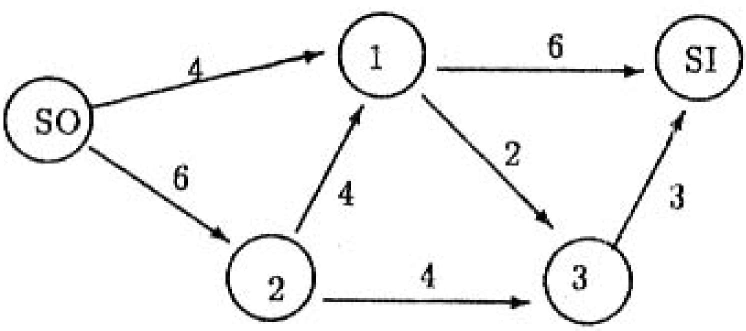


Figure 1: A Network Flow

考試科目 Course	線性代數	開課系級 Dept. & Class	應數	日期 Date, Period	月 日 第 節	試 C

(第一題至第七題, 每題 12 分. 第八題 16 分)

1. 設 T 爲由 R^4 映至 R^3 的線性變換

$$T(x_1,x_2,x_3,x_4)=(x_1,x_2,x_3,x_4)\begin{bmatrix}0&1&1\\1&0&1\\-1&1&0\\2&-1&1\end{bmatrix} \quad \forall(x_1,x_2,x_3,x_4)\in R^4$$

(a) T 的核(Kernel of T)之基底爲何 ?

(b) $T(R^4)$ 之基底爲何 ?

2. 設 T 爲由 R^3 映至 R^2 的線性變換且 $T(1,1,0)=(0,1)$, $T(0,1,1)=(-1,5)$, $T(1,0,1)=(1,2)$.

(a) $T(1,0,0)=?$

(b) 若 $T(1,p,q)=(1,0)$, 則 $(p,q)=?$

3. 設向量 $V_1=(0,1,1)$, $V_2=(1,2,0)$, $V_3=(3,-2,2)$.

(a) 由向量 V_1 和向量 V_2 所構成之平面 K 的方程式爲何 ?

(b) 向量 V_3 在平面 K 上之垂直投影向量爲何 ?

4. 設 $W_1=\{(x_1,x_2,x_3,x_4,x_5)\in R^5 \mid x_1-x_3-x_4=0\}$

$W_2=\{(x_1,x_2,x_3,x_4,x_5)\in R^5 \mid x_2=x_3=x_4\}$, 則

(a) $W_1\cap W_2=?$

(b) $\dim(W_1\cap W_2)=?$

5. 若 $A^2=\begin{bmatrix}1&1&0\\0&1&2\\0&0&1\end{bmatrix}$, 則

(a) $A=?$

(b) $A^{100}=?$

6. 設 $A=\begin{bmatrix}1&-1&-1\\-1&1&-1\\-1&-1&1\end{bmatrix}$,

(a) 若 B 是與 A 相似的對角矩陣, 則 $B=?$

(b) 若 C 爲一正交矩陣且滿足 $C^{-1}AC=B$, 則 $C=?$