考試科目	線性代數	所 別	應用數學系	考試時期	4月17日 星期六	下午第二節

1. Let $P_2 = \{ax^2 + bx + c \mid a,b,c \in \Re\}$ and $T: P_2 \to P_2$ be a mapping defined by $T(ax^2 + bx + c) = 2cx^2 - ax + b$ for all $ax^2 + bx + c \in P_2$.
   (a) Show that T is a linear transformation. (5%)
   (b) Is T an isomorphism of P_2? (5%)
   (c) Find the matrix representation $[T]_\alpha$ of T with respect to the ordered basis $\alpha = \{x^2, x, 1\}$. (5%)
   (d) Find the determinant of T. (5%)

2. Let S be the vector subspace in $\Re^4$ spanned by $\{(1,0,1,0),(0,1,0,1)\}$ and $v = (1,2,3,4) \in \Re^4$.
   (a) Is $v \in S$? (5%)
   (b) Find the orthogonal projection of v onto S. (10%)
   (c) Find the minimum distance from v to S. (10%)

3. (a) Let A be a nonsingular (invertible) $n \times n$ real matrix. Show that the inverse matrix A^{-1} of A can be expressed as a polynomial in A, that is,

$$A^{-1} = a_k A^k + a_{k-1} A^{k-1} + \cdots + a_1 A + a_0 I,,$$

   for some positive integer k and $a_0, \cdots, a_k \in \Re$. (10%)

   (b) Let $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.

   Express A^{-1} as a polynomial in A. (5%)

4. Let A be an $n \times n$ square matrix of rank one. Show that $A^2 = \alpha A$ for some $\alpha \in \Re$. (10%)

5. Let A be an $n \times n$ real symmetric matrix. Show that
   (a) All eigenvalues of A are real. (5%)
   (b) Eigenvectors of A corresponding to distinct eigenvalues are orthogonal. (5%)
   (c) A is positive definite if and only if $A = B^T B$ for some nonsingular matrix B. (10%)

6. Identify the conic $x^2 + 4xy + y^2 + 3x + y - 1 = 0$ and transform the conic into standard form. (10%)

備考	試題隨卷繳交

命題老師： （簽章）93年3月30日

考試科目	微積分	所別	應用數學系	考試時期	4月17日 星期六	下午第一節

1. (a) Evaluate the integral $\iint_{R_a} e^{-(x^2+y^2)}dxdy$, where a is a positive constant and

   $R_a = \{(x,y) \in \Re^2 \mid x^2 + y^2 \le a^2\}$. (8%)

   (b) Use (a) to evaluate the improper integral $\int_{-\infty}^{\infty} e^{-x^2} dx$. (8%)

2. Let $f: \Re \to \Re$ be a differentiable function such that $0 \le f'(x) \le f(x)$, $\forall x \in \Re$. Show that

   (a) the function $g(x) = e^{-x} f(x)$ is nonincreasing. (8%)

   (b) if f vanishes at some point, then $f \equiv 0$ on $\Re$. (8%)

3. Let $\{a_n\}$ be a real sequence. Prove or disprove the following statements:

   (a) If $\sum |a_n|$ converges, then $\sum a_n^2$ converges. (8%)

   (b) If $\sum a_n^2$ converges, then $\sum |a_n|$ converges. (8%)

4. (a) State the mean-value theorem for derivatives. (8%)

   (b) Use (a) to deduce the inequality $|\cos x - \cos y| \le |x - y|$, $\forall x, y \in \Re$. (8%)

5. For each $n = 1, 2, \cdots$, let $f_n(x) = x^n$, $0 \le x \le 1$.

   (a) Prove that the sequence $\{f_n\}$ converges pointwise on $[0,1]$. (8%)

   (b) Does $\{f_n\}$ converge uniformly on $[0,1]$? Justify your answer. (8%)

6. Use Lagrange's multiplier to prove that the minimum distance from a point (x_0, y_0, z_0) to a plane $ax + by + cz + d = 0$ is $\dfrac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$. (10%)

7. Evaluate the line integral $\oint_C xy^3 dx + 2x^2 y^2 dy$, where C denotes the boundary of the region in the first quadrant enclosed by the x-axis, the line $x = 1$, and the curve $y = x^3$. (10%)

備考	試題隨卷繳交

命題老師： （簽章） 92 年 3 月 21 日