考試科目	微積分	所別 8111 8116	應用數學系	考試時間	3月17日 星期六	第一節

1. Let $\sum_{n=1}^{\infty} a_n$ be a convergent series with sum s. Show that the series $\sum_{n=1}^{\infty}(a_n + 2a_{n+1})$ converges and find its sum. (20%)

2. Let $\vec{F}(x,y,z) = (x^3, y^3, z^3)$ be a vector field in $\mathfrak{R}^3$. Evaluate the surface integral $\iint_{S^2} \vec{F} \cdot \vec{n}\, dS$, where $S^2 = \{(x,y,z) \mid x^2 + y^2 + z^2 = 1\}$ is the unit sphere in $\mathfrak{R}^3$ and $\vec{n}$ is the unit outward normal vector field on S^2. (20%)

3. Find the maximum value of $f(x_1, \cdots, x_n) = x_1 + \cdots + x_n$ subject to the constraint ${x_1}^2 + \cdots + {x_n}^2 = 1$ and verify the inequality

$$\frac{x_1 + \cdots + x_n}{n} \le \left(\frac{{x_1}^2 + \cdots + {x_n}^2}{n}\right)^{1/2}.$$ (20%)

4. Let

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x} & \text{if } x \ne 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Show that f is differentiable on $\mathfrak{R}$, but f' is not continuous at $x = 0$. (20%)

5. Let f be a continuous function on $[1/2, 2]$. Evaluate the integral

$$\int_{1/2}^{2} \left(1 - \frac{1}{t^2}\right) f\left(t + \frac{1}{t}\right) dt.$$ (10%)

6. Does there exist a nonconstant continuous real-valued function f on $[0,1]$ which assumes only integer values? (10%)

備考	試題隨卷繳交
命題委員：	055 （簽章） 96年3月2日

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2.書寫時請勿超出格外，以免印製不清。
3.試題由郵寄遞者請以掛號寄出，以免遺失而示慎重。

國立政治大學九十六學年度研究所博碩士班入學考試命題紙 第1頁，共1頁

考試科目	線性代數	8111.8116 所別	應用數學	考試時間	3月17日 星期六 第二節

注意事項

1. 請儘量完整回答會寫的問題, 這會比每題都只做一小部份得到較高成績。
2. 請將理由陳述清楚, 引用定理請說明用到的定理內容, 如果答案太短可能需要提供該定理的證明。

Problem 1. (10 pts) Prove or give a counterexample. Let A be an $n \times n$ real symmetric matrix. For any column vectors $\mathbf{x}, \mathbf{y}$ in $\mathbb{R}^n$, define

$$\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{y}^T A x.$$

Then $\langle\, , \rangle$ is an inner product on $\mathbb{R}^n$.

Problem 2. (10 pts) Prove or give a counterexample. All 3×3 real matrix has a corresponding Jordan Canonical Form.

Problem 3. (20 pts) Let A be an $n \times n$ Hermitian matrix, that is, $A_{ij} = \overline{A_{ji}}$. Prove the following statements.

(1) All eigenvalues of A must be real.
(2) Eigenvectors corresponding to different eigenvalues are orthogonal.

Problem 4. (20 pts) Let $P_2(\mathbb{R})$ be the vector space of real polynomials of degree at most two. Define an inner product on $P_2(\mathbb{R})$ by

$$\langle f(x), g(x) \rangle = \int_{-1}^{1} f(t)g(t)\, dt.$$

Suppose $S = \text{span}\{1, x\}$. If $\|\cdot\|$ is the norm induced by the inner product $\langle\, , \rangle$, find all $h(x)$ in S such that

$$\|h(x) - x^2\|$$

is minimal. Justify your answer.

Problem 5. (20 pts) Let A be an $n \times n$ real matrix, where n is an even positive integer. If $AB = BA$ for all $n \times n$ real matrix B, show that $\det(A) \geq 0$.

Problem 6. (20 pts) Let $P_1(\mathbb{R})$ be the vector space of real polynomials of degree at most one. Suppose $T\colon P_1(\mathbb{R}) \to P_1(\mathbb{R})$ is a linear transformation defined by $T(a + bx) = 5a + 2b + (a + 4b)x$. Find $T^{100}(a + bx)$.

備考	試題隨卷繳交

命題委員： 056 （簽章） 年 月 日

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