考試科目 Course	數學(微積分)	開課系級 Dept. & Class	統計學系	日期 Date, Period	月 日 第 節	試 Co

(12 分) 1. Determine whether or not the function $f(x)$ is continuous and differentiable at the indicated points.

$$f(x)=\begin{cases} -x^2 & \text{if } x \text{ is not an integer and } x<0 \\ x^2 & \text{if } x \text{ is not an integer and } x>0 \\ 1 & \text{if } x \text{ is an integer} \end{cases}$$

(a) At $x=-1$

(b) At $x=0$

(c) At $x=1$

(10 分) 2. Calculate $\lim\limits_{n\to\infty}\frac{5}{n}\cdot\sum_{i=1}^{n}\left[\left(\frac{3i}{n}\right)^4-16\right]$.

(10 分) 3. Let $\{u_n\}$ be a sequence of real numbers such that $\sum_{j=1}^{n}u_j=\frac{n}{2(n+1)}$.

Calculate u_{10}.

(8 分) 4. Let $F(x)=f(x)\int_0^{x^2}f(y)\,dy$, where f is a differentiable function such that

(i) $\int_0^4 f(y)\,dy=1$, (ii) $f(2)=2$, (iii) $f(4)=3$, and (iv) $f'(2)=4$.

Calculate $F'(2)$.

(10 分) 5. Suppose that $f(x)\ge g(x)\ge k$ on $[a, b]$.
Let Ω be the region between the graph of $f(x)$ and $g(x)$ for $a\le x\le b$.
Find the volume of the solid which is generated by revolving Ω around the line $y=k$.

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6. The eigenvalues of $A = A^T$ (4×4 matrix) are $-1, -1, 2, 3$.

① $\text{rank}(A^TA) = ?$ why ? ② Is A diagonalizable ? why ?

③ Is A positive definite ? why ? (15%)

7. ① Find a basis for the subspace $V = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \middle| \; c = a+b,\ d = a-b \right\}$ of R^4.

② Is $V = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \middle| \; a-b=4 \right\}$ a subspace of R^3 ? why ?

③ $\left\{ \begin{bmatrix} 1 \\ 1 \\ c \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} \right\}$ is a linearly dependent set. Find $c = ?$ (15%)

8. Let $L : R^3 \longrightarrow R^3$ be the linear transformation defined by $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -x+2y \\ x+y+z \\ 2x-y+z \end{bmatrix}$

Is $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ in range L ? why ? (10%)

9. $V = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix} \right\}$.

① Find the projection of $\begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}$ on V.

② Find the distance from $\begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}$ to V. (10%)

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I. (20%) (複選題,選出正確答案即可,不需額外說明)

(1) The expected value for a sum is 50, with an standard error of 5. The chance process generating the sum is repeated 10 times. Which of the following is the sequence of observed values?
   (a) 53, 41, 48, 52, 57, 61, 58, 57, 51, 48
   (b) 47, 49, 50, 53, 47, 48, 50, 52, 51, 49
   (c) 45, 55, 50, 45, 50, 55, 50, 45, 55, 45

(2) A box contains one red marble, one green, and one blue. Someone reaches in with his left hand, and draws a marble at random; then, he reaches in with his right hand, and draws a second marble; the draws are made without replacement. Which of the following statements are true?
   (a) With probability 1/3, he gets the red marble in his left hand.
   (b) With probability 1/3, he gets the green marble in his right hand.
   (c) With probability 1/9, he gets the red marble in his left hand, and the green marble in his right hand.

(3) A 99% confidence interval for the proportion of drivers who regularly use seat belts is (0.27, 0.31).
   (a) In repeated sampling, approximately 99% of the intervals computed would contain the true proportion of drivers who regularly use seat belts.
   (b) There is sufficient evidence to reject the claim that 1/3 of all drivers regularly use seat belts at the 99% confidence level.
   (c) If the experiment is repeated a very large number of times, the true value of p will be between 0.27 and 0.31 approximately 99% of the time.

(4) Which of the following statements are true?
   (a) If the observed significance level (or P-value) is 1%, there is only 1 chance in 100 for the null hypothesis to be right.
   (b) If the observed significance level (or P-value) is 4%, then the result is statistically significant.
   (c) The P-value is data-dependent.

(5) What must be true if a sample statistic T is to be the minimum variance unbiased estimator of a parameter δ.
   (a) For large samples, T must be equal to δ.
   (b) T must have a smaller variance than any other estimator of δ.
   (c) None of the above is true.

II. (20%) True or False

(1) A statistically significant result cannot possibly be explained by chance.
(2) A difference that is highly statistically significant must be very important.
(3) A coin is tossed 500 times. The expected value for the difference between the percentage of heads in the first 400 tosses and the percentage of heads in the last 100 tosses equals 0.
(4) A coin is tossed 100 times and it lands heads 50 times, we can then conclude that the chance of getting heads on any toss equal to 50%.
(5) A coin is tossed 100 times, landing heads 53 times. However, the last seven tossed are all heads. The chance that the next toss will be heads is somewhat less than 50%.

(6) Two draws are made at random without replacement from a box containing 3 tickets. One of them has number 10 written on it. The other two have numbers 20, and 30, respectively, written on them. The first ticket is lost, and nobody knows what was written on it. In this case the two draws would be independent.

(7) A sample statistic T is said to be an unbiased estimator of a parameter θ if T equals θ for every possible sample.

(8) If the correlation coefficient is 0, it is still possible that the two variables are related in some way.

(9) If men always married women who were 4 years younger, the ages of husbands and wives was positively correlated.

(10) When the null hypothesis is not rejected, we can safely conclude that the hypothesis is true.

III. (10%) The random variable X, with distribution depending on parameter θ, takes on values in the set 0, 1, 2, 3,.... Suppose that it satisfies the recursion relationship

$$P(X=x)=\frac{\theta}{x}P(X=x-1) \quad \text{for } x=1,2,3,\ldots$$

(a) Find the density of X.

(b) Let X, Y and Z be i.i.d. random variables with density you found in (a). Find $P(X=k\mid X+Y+Z=n)$. What is the conditional distribution of X given $X+Y+Z$?

IV. (10%)

(a) A coin will be tossed, and you win a dollar if the number of heads is exactly equal to the number of tails. Which is better for you: 10 tosses or 100 tosses? Explain.

(b) A coin will be tossed, and you win a dollar if the percentage of heads is between 40% and 60%. Which is better for you: 10 tosses or 100 tosses? Explain.

(c) A coin will be tossed, and you win a dollar if there are more than 60% heads. Which is better for you: 10 tosses or 100 tosses? Explain.

V. (40%) Let $X_1, X_2, \ldots, X_n$ be independent, X_i exponentially distributed with expectation $i\theta$, i.e. $f(x_i; i\theta)=\frac{1}{i\theta}\exp(-\frac{x_i}{i\theta})$, $i=1,2,\ldots,n$.

(a) Find the m.l.e. $\hat{\theta}$ for θ.

(b) Show that $2n\hat{\theta}/\theta$ has a chi-square distribution with $2n$ degrees of freedom.

(c) Find a complete sufficient statistic for this model.

(d) Show that $\hat{\theta}$ is consistent.

(e) Find the minimum variance unbiased estimator (MVUE) of θ.

(f) Show that the MVUE of θ is efficient.

(g) Find a $(1-\alpha)$ confidence interval for θ and a $(1-\alpha)$ confidence interval for $1/\theta$.

(h) Find the UMP (uniformly most powerful) size α test for testing that $\theta=2$ against $\theta<2$.

$1/\theta$.

(h) Find the UMP (uniformly most powerful) size α test for testing that $\theta=2$ against $\theta<2$.