考試科目 Course	經濟理論	系級	金融所	日期 Date, Period	6月10日 第1節	試題編號 CourseNo.

I. Overlapping Generation Model 與 Ricardian Equivalence (25 分)

考慮下面的動態經濟體系：

- 時間 $t=0,1,2,\ldots$，每一期的人口數為 L_t，$L_t=(1+n)L_{t+1}$，即人口成長率為 n。
- 每人只活兩期，其效用函數為 $U_t=\dfrac{C_{1t}^{1-\theta}}{1-\theta}+\dfrac{1}{1+\rho}\dfrac{C_{2t}^{1-\theta}}{1-\theta}$，下標 t 代表第 t 期出生的人(即第 t 代)，其中 $\theta>0$，$\rho>-1$，C_{1t} 及 C_{2t} 分別為年輕時及年老時的消費。在 $t\geq 1$ 期出生的每一個人，其唯一的要素稟賦是為年輕時的一單位勞動。
- 在 $t=0$ 時有一批年老者，擁有 K_0 的資本存量。
- 僅有的財貨由很多廠商生產，生產函數為 $Y_t=A_tL_tk_t^{\alpha}$ 其中 $k_t = K_t/(A_t L_t)$，$0<\alpha<1$。
- 外生技術成長率為 g，反應在 A 的變化上，$A_{t+1} = (1+g)A_t$。
- 當期沒有被消費的財貨，可以被留到下一期作為生產投入 --- 資本 (K)。
- 假設所有的市場均為完全競爭的市場。

令 w_t 及 r_t 別為第 t 期的勞動價格及資本報酬率。請回答下列問題：

(1) (5 分) 定義均衡。(定義你所用的每一個符號，否則不予給分)

(2) (5 分) 請導出儲蓄率與資本報酬率的關係。

(3) (5 分) 請導出在 $\theta=1$ 下的 k_t 與 k_{t+1} 的均衡關係式。

在上述的模型中導入政府部門，政府部門的消費為 G。

(4) (10 分) 政府可以用稅收或發行債券的方式來融通其支出。請問這種不同的融通方式對均衡的影響一樣嗎？為什麼？

II. Financial Crisis (25 分)

In the Tequila Crisis and the Asia Crisis there was widespread contagion to several emerging markets. Did these crises spread in a purely random fashion, or is there a set of fundamentals that helps to explain the spread of each crisis? If the latter is true, did the contagion follow the same pattern in both episodes, or was each of a different nature? Explain your answer.

III. Pareto Efficiency and Competitive Equilibrium (20 分)

有兩種財貨 x 和 y，有甲、乙二人，其效用函數分別為 $U_甲=\min\{2x, y\}$ 及 $U_乙= xy$。

(1) (7 分) 有 10 單位的 x 財及 10 單位的 y 財分配給甲、乙二人。請在 Edgeworth Box 中標出所有的 Pareto 效率配置點 (Pareto efficient allocation)。Edgeworth Box 的左下方原點需為甲的原點，右上方的原點為乙的原點，水平軸為 x 財，垂直軸為 y 財。

(2) (3 分) 如果甲擁有 8 單位的 x 財及 2 單位的 y 財，乙擁有 2 單位的 x 財及 8 單位的 y 財，此二人間會有交易嗎？如果有，交易的價格會落於什麼範圍內？

(3) (3 分) 如果 x 及 y 的消費量需為整數，則有那些可能的交易價格?

(4) (7 分)如果有 1000 個甲及 1000 個乙，假設每個人單獨都無法影響市場價格；財貨的消費單位可以細微分割。請計算完全競爭的均衡 (competitive equilibrium)。(先定義均衡，再計算。)

IV. Imperfect Information and Adverse Selection (30 分)

Suppose that there are two types of individuals: high-risk types, for whom the accident probability of loss is π_h, and low-risk, for whom the accident probability is $\pi_1 < \pi_h$. Both types are otherwise identical: they have the same concave utility function $v(.)$, the same endowed income y and suffer the same loss L. The probability of accident is exogenous and is known to the individual. The proportion of the population who are low risk is γ. Insurers also know v, y, L, π_h, π_1 and γ, but the insurer cannot tell high-risk types from low-risk types. Denote the insurance contracts for both types of individuals by (P_h, q_h) and (P_1, q_1) respectively. P is the insurance

考試科目 Course	高等微積分	開課系級 Dept. & Class	金融系博士班	日期 Date, Period	六月十日 第二節	試題編號 Course No.

1. (20%) Let D be an $N \times K$ matrix, g and θ are vectors in $\mathbb{R}^N$. Consider the following equation:

(E) $\quad D\psi = g$, where $\psi \in \mathbb{R}^K_{++}$ (i.e. $\psi = [\psi_1 \cdots \psi_K]'$, $\psi_k > 0$, $\forall k$)

Define the following condition as (A):

(A): there exists $\theta \in \mathbb{R}^N$ such that $g \cdot \theta \leq 0$ and $D'\theta > 0$,
or there exists $\theta \in \mathbb{R}^N$ such that $g \cdot \theta < 0$ and $D'\theta \geq 0$.

Show that (E) has a solution ψ if and only if (A) doesn't hold.

2. (20%) Following question 1, define $\text{span}(D) = \{ D'\theta : \theta \in \mathbb{R}^N \}$

Show that $\text{span}(D) = \mathbb{R}^K$ if and only if the solution in (E) is unique.

3. (20%)

(a) (10%) Suppose $E \subset \mathbb{R}^n$ is Lebesgue measurable. Let $(f_n)_{n=1}^{\infty}$ be a sequence of measurable functions such that $f_n(x) \to f(x)$, $\forall x \in E$, as $n \to \infty$. Let g be a Lebesgue integrable function on E and

$|f_n(x)| \leq g(x)$, $\forall n$, $\forall x \in E$.

Show that $\lim_{n\to\infty} \int_E f_n \, d\mu = \int_E f \, d\mu$

(b) (10%) Show that if $\mu(E) < \infty$, $(f_n)_{n=1}^{\infty}$ is uniformly bounded on E and $f_n(x) \to f(x)$ $\forall x \in E$, then $\lim_{n\to\infty} \int_E f_n \, d\mu = \int_E f \, d\mu$.

4. (20%) Show that every finite-dimensional inner product space has an orthonormal basis.

5. (20%)

(a) (10%) $f : X \to Y$, where X, Y are metric spaces, show that f is continuous on X if and only if $f^{-1}(V)$ is open in X for any open set V in Y.

(b) (10%) $f : X \to Y$ is continuous mapping. Suppose X is compact, show that $f(X)$ is compact.

考試科目 Course	計量經濟理論	所別	博士班	考試時間 Date, Period	第 二 節	Course No.

(10%) 1. Show that the rank of a symmetric idempotent matrix is equal to its trace.

(10%) 2. Show that convergence in mean square implies convergence in probability.

(20%) 3. Consider the linear regression model:

$$Y = X\beta + u \quad \text{or} \quad y_t = x_t'\beta + u_t, \quad t = 1, 2, \cdots T.$$

where Y = $T\times 1$ vector; X = $T\times K$ matrix; β = $K\times 1$ vector, u = $T\times 1$ vector. and $Eu = 0$, $Euu' = \sigma^2\Omega$. Suppose we have L instruments such that $E(h_t u_t) = 0$, where $h_t = [h_{1t} \cdots h_{Lt}]'$ is an $L\times 1$ vector of instruments. Derive the 2SLS and GMM estimators for each case of $L = K$ and $L > K$.

(20%) 4. Given the following model:

$$y_t = x_t'\beta + \varepsilon_t \;;\quad \varepsilon_t = \rho\varepsilon_{t-1} + u_t - \lambda u_{t-1} \;;$$

$$E u_t = 0, \quad E u_t^2 = \sigma_u^2, \quad \operatorname{cov}(u_t, u_s) = 0, \text{ for } t \neq s.$$

(15%) a) Derive the covariance matrix of the disturbances.

(5%) b) What parameter is estimated by the regression of OLS residuals on their lagged values?

(20%) 5. In the regression model: $Y = X\beta + u$; $Eu = 0$, $Euu' = \Omega$ and the dimension of vectors or matrix are as in question 3. Let $\hat{\beta}$ is the GLS estimator of β. How do we test $H_0: R\beta = g$, where R is a $J\times K$ matrix, g is $J\times 1$ vector, according to:

(10%) a) Ω is known and the disturbances are normally distributed.

(10%) b) Ω is known but the disturbances are not normally distributed.

(20%) 6. Explain the following questions by specifying explicit models:

(10%)(a) GARCH and GARCH-in-mean models

(10%)(b) Unit root in economic time series.

考試科目 Course	貨幣銀行	系級	金融	日期 Date. Period	6 月 10 日 第 二 節

(25%) 1. 說明重貼現率的調高是否一定代表央行緊縮貨幣政策？

(25%) 2. 請先說明良好貨幣政策中間目標的條件。其次，如果 $\dot{Y}^e$ =6%，$\dot{P}^e$ =2%，且某人估計貨幣需求函數為

$$\ln M^d = 1.0 + 0.9 \ln y - 0.1 i$$

則央行的貨幣目標區的中間值為？

(25%) 3. 說明我國的「工業銀行」與日本、德國的銀行體制的差別。

(25%) 4. 說明下列二個觀念的異同：BIS 資本適足率的「風險性資產」，與貨幣總計數的「Divisia 貨幣」異同。

考試科目 Course	財務理論	系級 金融系 博士入學考試	日期 Date, Period	6月10日 第2節	試題編號 Course No.

1. Explain the theory of optimal capital structure in the presence of bankruptcy costs. (25分)。

2. What is Arbitrage Pricing Theory (APT)? Using a two-factor model as a case, derive the theory with economic intuitions. At least, you should be able to explain the economic intuitions of how the theory is derived. (25分)。

3. Compare the differences between APT and the CAPM. There are seven notable features you can make comparisons between these two theories. (25分)。

4. Why is the option pricing theory a risk-neutral valuation theory? Can you prove it analytically? If you can, you are very good. (25分)。 At least, you should be able to write down the economic intuitions of risk-neutral valuation.