

考試科目	經濟學原理	系別	經濟學系	考試時間	2月10日(日) 下午第2節
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1. 試以圖形表示總產出、平均產出、及邊際產出曲線三者之間的關係，並說明合理的生產階段應落於何處（或那一生產階段）？（25分）
2. 試以鉅結需求曲線（kinked demand curve）圖形說明寡佔市場的價格僵固定。（25分）
3. 貨幣供給增加如何影響經濟活動（國民產出），以圖形說明凱因斯學派的論點。（25分）
4. 以家計、企業兩部內，產品、要素兩市場，圖形表示總體經濟循環（macroeconomic circular flow），並據以說明關支法所得法的國民（內）生產毛額的計算（包括那些項目）？（25分）

試科目	微積分	系別	經濟學系	考試時間	星期	月	日	上午	下午
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- (5%) Find the total derivative  $dz/dy$ , given the function  $z = f(x, y) = 2x + xy - y^2$  where  $y = \sqrt{x/3}$ .
- (5%) Find the rate of change of output with respect to time, if the production function is  $Q = A(t)K^a L^b$ , where  $A(t)$  is an increasing function of  $t$ , and  $K = K_0 + at$ , and  $L = L_0 + bt$ .
- (5%) If the equation  $F(x, y, z) = 0$  implicitly defines each of the three variables as a function of the other two variables, and if all the derivatives in question exist, find the value of  $\frac{\partial z}{\partial x} \frac{\partial x}{\partial y} \frac{\partial y}{\partial z}$ .
- (10%) Given  $x^2 + 3xy + 2yz + y^2 + z^2 - 11 = 0$ , is an implicit function  $z = f(x, y)$  defined around the point  $(x=1, y=2, z=0)$ ? If so, find  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$  by the implicit-function rule, and evaluate them at that point.
- (10%) Given the function  $\phi(x) = e^{2x}$ :
  - Write the polynomial part  $P_n$  of its Maclaurin series.
  - Write the Lagrange form of the remainder  $R_n$ . Determine whether  $R_n \rightarrow 0$  as  $n \rightarrow \infty$ , that is, whether the series is convergent to  $\phi(x)$ .
  - If convergent, so that  $\phi(x)$  may be expressed as an infinite series, write out this series.
- (10%) Use the Lagrange-multiplier method to find the stationary values of  $z = x^2 + 2xy + w^2$ , subject to  $2x + y + w^2 = 24$  and  $x + w = 8$ .
- (10%) Evaluate the following:
  - $\lim_{x \rightarrow 0^+} 2x^x$  (5%)
  - $\lim_{x \rightarrow \infty} \frac{5^x - e^x}{x}$  (5%)
- (10%) Evaluate the following:
  - $\int_{-1}^{e-2} \frac{dx}{x+2}$  (5%)
  - $\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} \sqrt{1 - \cos^2 x} dx$  (5%)

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9. (10%) A local business firm is planning to advertise a special anniversary sale on radio and television during a particular week. For that, a maximum budget of \$16,000 is approved. It is found that radio commercials cost \$800 per 30-second spot (call this  $X_1$ ), with a minimum contract of five spots. Television commercials, on the other hand, cost \$4000 per spot (call this  $X_2$ ). Because of heavy demand, however, only four television spots are still available in the designated week. On the basis of estimated size of audience and other factors, it is believed that a television spot is six times as effective as a radio spot in reaching potential customers. How should the firm allocate its advertising to attract the largest possible number of potential customers?

10. (10%) Let  $Q$  be output level and  $h$ ,  $j$  and  $k$  be parameters. A quadratic profit function  $\pi(Q) = hQ^2 + jQ + k$  is to be used to reflect the following assumptions:

- (a) If nothing is produced, the profit will be negative (because of fixed costs).
- (b) The profit function is strictly concave.
- (c) The maximum profit occurs at a positive output level  $\bar{Q}$ .

What restrictions are called for the parameters  $h$ ,  $j$  and  $k$ ?

11. (5%) Transform  $t = \log_8^{3y}$  to its natural logarithmic form.

12. (10%) Suppose the value of timber (already planted on some given land) is the following increasing function of time:

$$V = 2^{\sqrt{t}}$$

expressed in units of \$1000. Assuming a discount rate of  $r$  (on the continuous basis) and also assuming zero upkeep cost during the period of timber growth, what is the optimal time to cut the timber for sale?