

考試科目	微積分(一)	系 別	應用數學系	考試時間	7月10日上午第2節 星期六
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1. (15%) Prove that $\lim_{x \rightarrow 0} \sin \frac{1}{x}$ does not exist at $x=0$.

2. (15%) Show that if f is strictly increasing on an interval I where it is differentiable, then $f'(x) \geq 0$ for all x in I .

3. (20%) Find all points on the curve of

$$(x^2 + y^2)^2 = 4(x^2 - y^2)$$
 where the tangent is horizontal.

4. (20%) Find all relative extrema and saddle points on the graph of $f(x, y) = x^2 y^4$.

5. (30%) For A and B positive constants, define

$$f(x) = (e^x + Ax)^{\frac{B}{x}}$$

(1) Compute $L_1 = \lim_{x \rightarrow 0} f(x)$ and $L_2 = \lim_{x \rightarrow +\infty} f(x)$.

(2) What is the largest value of A for which the equation $L_1 = BL_2$ has a solution? What are L_1 and L_2 in this case?

第1題至第16題為選擇題，請選最接近或最適當的唯一選項。其中第1到第10題，每題4分。第11到第16題，每題5分。選擇題僅需寫上題号及選項，答錯不倒扣。第17題到第19題為計算題，必須有計算過程，每題10分。須把每題題号，於答案紙上寫清楚。

A. 選擇題(一) (每題4分，不倒扣)

- Find an expression equivalent to $\int [f(x)g(x) + 6h(x) - g(x)] dx$.
 (a) $\int f(x)g(x) dx + 6 \int h(x) dx - \int g(x) dx$ (b) $\int [f(x)g(x) + 6h(x) - g(x)] dx$
 (c) $\int f(x)g(x) dx + 6 \int [h(x) - \frac{1}{6}g(x)] dx$ (d) None of a, b, or c. (e) All of a, b, and c.
- Consider $F(x) = \int_x^1 \sqrt{1+t^2} dt$. Find $F'(x)$.
 (a) $\sqrt{1+x^2}$ (b) $\frac{1}{\sqrt{2}} - \frac{x}{\sqrt{1+x^2}}$ (c) 1 (d) $-\sqrt{1+x^2}$ (e) None of these.
- Identify the definite integral that represents the arc length of the curve $y = \frac{1}{x}$ over $[1, 3]$.
 (a) $\int_1^3 \sqrt{1+(1/x)^2} dx$ (b) $\int_1^3 \sqrt{1+x^2} dx$ (c) $\int_1^3 \sqrt{1+\frac{1}{x^2}} dx$ (d) $\int_1^3 \sqrt{(\frac{1}{x})+(\frac{1}{x^2})} dx$ (e) None of these.
- Use the fact that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ to find $\sum_{n=3}^{\infty} \frac{1}{n^2}$.
 (a) $\frac{\pi^2}{6}$ (b) $\frac{\pi^2-12}{6}$ (c) $\frac{\pi^2-6}{6}$ (d) $\frac{2\pi^2-15}{12}$ (e) None of these.
- Determine the convergence or divergence of $\sum_{n=1}^{\infty} \frac{9n^2+4}{n^2-n}$ using the Limit Comparison Test (LCT).
 (a) Converges by LCT. (b) Diverges by LCT. (c) LCT does not apply (d) LCT fails (e) None of these.
- Determine whether $\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n}$ converges. If it does, classify it as absolutely convergent or conditionally convergent.
 (a) Divergent. (b) Absolutely convergent (c) Conditionally convergent (d) None of these.
- Investigate $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{n!}$ for convergence or divergence.
 (a) Diverges by Ratio Test. (b) Converges by n-th-Term Test (c) Diverges by Root Test (d) Converges by Integral Test.
- Evaluate the integral $\int_0^1 \int_0^x \int_0^{xy} xyz dz dy dx$.
 (a) $\frac{1}{32}$ (b) $\frac{1}{48}$ (c) $\frac{1}{64}$ (d) $\frac{1}{96}$ (e) None of these.
- Evaluate: $\int_0^1 (e^{-t} \underline{i} + \frac{1}{t+1} \underline{j}) dt$
 (a) $-\frac{1}{2} \underline{i} + \ln 2 \underline{j}$ (b) $(1 - \frac{1}{e}) \underline{i} + \ln 2 \underline{j}$ (c) $(1 - e) \underline{i} + \ln 1 \underline{j}$ (d) $\frac{1}{2} \underline{i} + \ln 2 \underline{j}$ (e) None of these.
- Evaluate the definite integral: $\int_{15}^{20} \frac{x}{\sqrt{25-x^2}} dx$.
 (a) $\sqrt{15} - 2\sqrt{5}$ (b) 5 (c) -5 (d) $2\sqrt{5} - \sqrt{15}$ (e) None of these.

B. 選擇題(二) (每題5分，不倒扣)

考試科目	微積分(二)	系別	應用數學系	考試時間	7月10日 下午第3節 星期六
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11. Evaluate the indefinite integral: $\int \frac{1}{x \cdot e^{5x}} dx$.
 (a) $\frac{1}{5}e^{5x} + C$ (b) $\frac{1}{5}x e^{5x} + C$ (c) $\frac{1}{5}e^{-5x} + C$ (d) $\frac{1}{5}x e^{-5x} + C$ (e) None of these.
12. Evaluate the indefinite integral: $\int \sinh(-6x) \cosh(-3x) dx$.
 (a) $-\frac{1}{3} \sinh^2(-3x) + C$ (b) $-\frac{1}{6} \sinh^2(-3x) + C$ (c) $-\frac{1}{3} \cosh^2(-3x) + C$ (d) $-\frac{2}{9} \cosh^2(-3x) + C$ (e) None of these.
13. Find the volume of the solid formed by revolving the region bounded by the graphs of $f(x) = \frac{3}{8} \sqrt{64-x^2}$ and $g(x) = \frac{3}{8} |x|$ about the x-axis.
 (a) 48π (b) $48\sqrt{2}\pi$ (c) $-24(8-5\sqrt{2})\pi$ (d) None of these
14. Evaluate the triple integral $\int_0^{2\pi} \int_0^{\pi/4} \int_0^1 \rho^2 \sin \phi d\rho d\phi d\theta$.
 (a) $\frac{(2-\sqrt{2})\pi}{4}$ (b) $\frac{(2-\sqrt{2})\pi}{2}$ (c) $\frac{(2-\sqrt{2})\pi}{8}$ (d) $\frac{1}{4}\pi$ (e) None of these.
15. Evaluate: $\int x \cos 2x dx$.
 (a) $\frac{1}{4} \sin 2x + \frac{1}{2} x \cos 2x + C$ (b) $\frac{1}{4} \cos 2x - \frac{1}{2} x \sin 2x + C$ (c) $\frac{1}{4} \cos 2x + \frac{1}{2} x \sin 2x + C$ (d) $\frac{1}{4} \sin 2x - \frac{1}{2} x \cos 2x + C$ (e) None of these
16. Evaluate: $\int \sin^3 x \cos^3 x dx$.
 (a) $\frac{\cos^3 x}{3} - \frac{\cos^5 x}{5} + C$ (b) $\frac{1}{12} \sin^9 x \cos^3 x + C$ (c) $\frac{1}{4} \sin^4 x - \frac{1}{8} \sin^6 x + C$ (d) $\frac{1}{5} \cos^5 x - \frac{1}{3} \cos^3 x + C$ (e) None of these.

C. 計算題 (每題10分, 需計算過程)

17. Consider $f(y) = -y^2 + 4$ and $g(y) = -3y$. (i) Sketch the region bounded by the graphs of f and g . (ii) Find the points of intersection of the graphs of f and g . (iii) Calculate the area of the region.
18. Consider the region bounded by the graphs of $y = \frac{1}{x}$, $y = x^2$, and $x = 2$. (i) Sketch the region described. (ii) Find the point(s) of intersection. (iii) Calculate the volume of the solid formed when this region is revolved about the line $x = 2$.
19. Evaluate $\iint_R e^{x^2+y^2} dy dx$, where R is the semicircular region bounded by the x-axis and the curve $y = \sqrt{1-x^2}$.