試科目	微積分(一)	系別	應用數學系 71	考試時間	七月五日(星期六)上午第

I . Prove or disprove the following statements

1. (15%) The limit $\lim_{x\to\infty}\left(\sqrt[n]{(x+a_1)\cdots(x+a_k)\cdots(x+a_n)}-x\right)$ exists for $a_k\in\mathbb{R}$.

2. (15%) $\frac{d^2}{dx^2}(\sin 2x) = -\sin(2x) \qquad \forall x\in\mathbb{R}$.

3. (20%) The series

$$\sum_{n=2}^{\infty}\frac{\cos n^p}{n(\ln n)^{\alpha}} \text{ and } \sum_{n=2}^{\infty}\frac{(\ln n)^{\beta}\cos n^p}{n^{\alpha}}$$

converge for every $\alpha>1$, $\beta\geq 1$and $p\geq 1$.

II. A farmer has 1000 feet of fence and wants to build a rectangular enclosure along a straight wall. If the side along the wall needs no fence.

4.(10%) Find the dimensions that make the enclosure as large as possible.

5.(10%) Find the maximum area.

III. The following series is a rearrangement of the alternating harmonic series in which there appear alternately three positive terms followed by two negative terms

$$1+\frac{1}{3}+\frac{1}{5}-\frac{1}{2}-\frac{1}{4}+\frac{1}{7}+\frac{1}{9}+\frac{1}{11}-\frac{1}{6}-\frac{1}{8}+++--\cdots$$

6.(15%) Show that this series converges and has sum $\ln\sqrt{6}$.

IV. Prove or disprove the statements:

For the sequence of functions $f_n(x)=(x^2-1)^n$, define $g_n(x)=f_n^{(3)}(x)$, $n\geq 1$, $g_0(x)=1$, then

7.(15%) $g_n(x)$ converges uniformly in $[0,\sqrt{2}]$.

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試 科 目	微積分(二)	系 別	应用數學系 71	考 試 時 間	七月五日星期六 下午第四

1. (40%) Find the folowing definite integrals :

   a.

$$\int_0^1 \left(\sqrt{1-t^2} + \frac{t}{1+t^4} \right) dt$$

   b.

$$\int_1^3 x^3 \ln x \, dx$$

   c.

$$\int_2^4 \frac{2x-1}{x^2-1} dx$$

   d.

$$\int_0^1 x^2 \sin 2x dx$$

2. (10%) Find the limit .

$$\lim_{n\to\infty} \int_0^\infty e^{-n^2 x} dx$$

3. (10%) Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for

$$f(x,y) = \int_{2x}^{y^2} \sin(t^2) dt$$

4. (10%) Evaluate the double integral

$$\iint_D \cos e^x dA$$

   where D is the region bounded by $y = e^x, y = -e^x, x = 0$ and $x = \ln 2$.

5. (10%) Evaluate the triple integral

$$\iiint_D e^z dV$$

   where $D = \{(x,y,z) \mid 0 \le x \le 1, 0 \le y \le x, 0 \le z \le x+y\}$.

6. (20%)

   a. Find the interval I of convergence of the series $\sum_{k=0}^{\infty} \left(\frac{x}{2}\right)^k$.

   b. Let $f(x) = \sum_{k=0}^{\infty} \left(\frac{x}{2}\right)^k$, find $\int_0^x f(t)dt$.