國立政治大學 九十三學年度轉學生入學考試

考試科目	微積分(一)	系別	應用數學系	考試時間	7月3日 星期六 (上)下午第二節

(Ten points for each problem)

1. Determine whether $\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{n+n}\right)$ exists or not.
2. Suppose that $\lim_{x\to a} f(x)$ exists and $\lim_{x\to a^-} f(x)=L_1$ and $\lim_{x\to a^+} f(x)=L_2$. Show that $L_1=L_2$.
3. Use definition to show that $\frac{d}{dx}e^x=e^x$.
4. Show that $\frac{d}{dx}\sin x=\cos x$.
5. Prove or disprove that if $f(x)$ is differentiable at $x=a$, then $f(x)$ is continuous at $x=a$.
6. Prove the Chain Rule.
7. Show that if $f(x)>0$ on (a,b), then $f(x)$ is increasing on (a,b).
8. Show that the minimum average cost occurs when the average cost function equals to the marginal cost function.
9. Solve for y if $\frac{d}{dx}y=ky(M-y)$ where k, M are constant.
10. Prove $\int_a^b f(x)dx=F(b)-F(a)$ where F is any antiderivative of f.

備考	試題隨卷繳交

命題委員 -174- (簽章) 93年 6月 25日

考試科目	微積分(二)	系別	應用數學系	考試時間	7月3日 星期六 下午第 2 節

注意：無演算過程之答案將不于記分

1. Evaluate the following iterated integrals. (5%+5%+10%)

   (a) $\int_0^1 \int_{y/2}^{1/2} e^{-x^2}\,dx dy$　　(b) $\int_0^{\ln 10} \int_{e^x}^{10} \frac{1}{\ln y}\,dy dx$

   (c) $\int_0^5 \int_0^{\sqrt{25-x^2}} \int_0^{\sqrt{25-x^2-y^2}} \frac{1}{1+x^2+y^2+z^2}\,dz dy dx$

2. Evaluate the following double or triple integrals. (30%)

   (a) $\iint_R \frac{y}{x^2+y^2}\,dA$, where R is the triangle bounded by $y=x,\ y=2x,\ x=2$.

   (b) $\iint_R \arctan\frac{y}{x}\,dA$, where R is the region bounded by $x^2+y^2 \ge 1,\ x^2+y^2 \le 4$, and $0 \le y \le x$.

   (c) $\iiint_E \sqrt{x^2+z^2}\,dV$, where E is the region bounded by the paraboloid $y=x^2+z^2$ and the plane $y=4$.

3. Find the area of the region common to the two regions bounded by $r=2+\cos\theta$ and $r=-3\cos\theta$. (10%)

4. Find the extreme values of $f(x,y)=x^4+y^4-4xy+1$ subject to the constraint $x^2+y^2 \le 2$. (15%)

5. Find the directional derivative of $f(x,y,z)=x\arctan yz$ at the point $P(4,1,1)$ in the direction from P to $Q(5,3,0)$. In which direction does the value of f increases most at P and what will be the maximum increase? (15%)

6. Find the volume of the solid region cut from the sphere $x^2+y^2+z^2=4$ by the cylinder $(x+1)^2+y^2=1$. (10%)

備考	試題隨卷繳交

命題委員：-175-　　（簽章）93年 6 月 23 日

命題紙使用說明：1.試題將用原件印製，敬請使用黑色墨水正楷書寫或打字（紅色不能製版請勿使用）。
2.書寫時請勿超出格外，以免印製不清。
3.試題由郵寄遞者請以掛號寄出，以免遺失而示慎重。