考試科目	微積分(一)	系別	應用數學	考試時間	7月7日 星期六 第二節

* Show all your work.

1. Prove that if θ is measured in radians, then $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ (10%)

2. Use the definition of derivative to find $f'(x)$ for the function $f(x) = 2/x$. (10%)

3. Find the derivative of y with respect to x, t or θ.

   (a) $x + \tan(xy) = 0$

   (b) $y = 2^{\sin 3t}$

   (c) $y = (\theta \cdot \sin\theta)/\sqrt{\sec\theta}$

   (d) $y = \log_5 \sqrt{\left(\frac{7x}{3x+2}\right)^{\ln 5}}$

   (5% each).

4. Graph the function $y = \frac{x^3+1}{x}$.

   (a) Find the equations of the asymptotes.

   (b) Determine rise and fall (increasing, decreasing)

   (c) Determine the concavity.

   (d) Sketch the graph. (5% each)

5. Find all local maxima, local minima, and saddle points of the function $f(x,y) = 6x^2 - 2x^3 + 3y^2 + 6xy$. (10%)

6. Find the points on the hyperbolic cylinder $x^2 - z^2 - 1 = 0$ closest to the origin. (10%) (*Use the Method of Lagrange Multiplier)

7. Water runs into a conical tank at the rate of $9\ \text{ft}^3/\text{min}$. The tank stands point down and has a height of 10 ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep? (see the figure below) (10%)

備考	試題隨卷繳交

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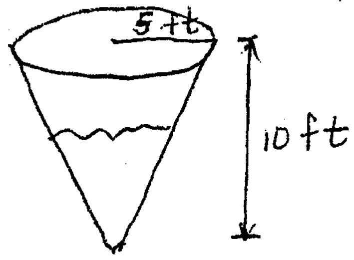

the conical tank in problem 7.

8. Show that
$$f(x,y)=\begin{cases}\dfrac{2xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$
is continuous at every point except the origin. (10%)

備考	試題隨卷繳交

國立政治大學九十六學年度轉學生入學考試命題紙

考試科目	微積分(二)	系別	應用數學系	考試時間	7月7日 星期六	第4節

(1) Compute the volume of the solid enclosed by the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, where a,b,c are positive constants. (15%)

(2) Let $f(x)$ and $g(x)$ be two continuous real-valued functions on $[a,b]$. Show that $\left|\int_a^b f(x)g(x)dx\right|^2 \le \left(\int_a^b |f(x)|^2 dx\right)\left(\int_a^b |g(x)|^2 dx\right)$. (15%)

(3) Evaluate the improper integral $\int_0^\infty xe^{-x}dx$ if exists. (10%)

(4) Find the area of the region enclosed by the curves $y=x$ and $y=x^3$ in the plane. (10%)

(5) Evaluate the following indefinite integrals

(a) $\int \ln x dx$ (b) $\int \sec x dx$. (20%)

(6) Find the following limits

(a) $\lim_{n\to\infty}\left(\int_0^1 e^{-nx^2}dx\right)^{1/n}$ if exists

(b) $\lim_{x\to 0^+} x\int_x^1 \frac{\cos t}{t^2}dt$ if exists.

(c) $\lim_{n\to\infty}\sum_{k=0}^{n-1}\frac{1}{\sqrt{n^2-k^2}}$ if exists. (30%)

備考	試題隨卷繳交