

考試科目	統計學	系所別	經濟學系	考試時間	2月18日(一)第三節
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注意事項:

- (1) 請依題號順序作答。
- (2) 不可使用計算機。
- (3) 答題若過程錯誤 (或沒有過程) 但答案正確, 將以「零分」計算。

1. (40%) The joint probability density function (pdf) of  $X$  and  $Y$  is given by

$$f_{X,Y}(x, y) = \begin{cases} c, & 0 \leq x \leq y \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

where  $c$  is an unknown constant.

- (1) (5%) Find the value of  $c$  that makes this a valid pdf.
- (2) (10%) Find the **marginal pdfs** for  $X$  and for  $Y$ .
- (3) (5%) Are the random variables  $X$  and  $Y$  **independent**? Why or why not?
- (4) (10%) Derive the **conditional mean** of  $Y$ , given  $X = x$ .
- (5) (5%) Derive the **conditional variance** of  $Y$ , given  $X = x$ .
- (6) (5%) Find  $\mathbb{P}(\frac{3}{4} \leq Y \leq \frac{7}{8} | X = \frac{1}{4})$ . (You should clearly write down the reason.)

2. (25%) Let  $X_1, X_2, \dots, X_n$  be a random sample of size  $n$  from the distribution with probability density function (pdf):

$$f_X(x; \delta) = k \cdot \delta x^{-1+\delta}, \quad 0 < x < 1, \quad 0 < \delta < \infty,$$

with an unknown parameter  $\delta$ . Then given a realization of data as  $x_1, x_2, \dots, x_n$ ,

- (1) (5%) Find the value of  $k$  that makes this a valid pdf.
- (2) (10%) Find the **method of moments estimator** and the corresponding **estimate** for  $\delta$ .
- (3) (10%) Find the **maximum likelihood estimator** the corresponding **estimate** for  $\delta$ .

備

註

- 一、作答於試題上者, 不予計分。
- 二、試題請隨卷繳交。

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3. (15%) Let  $X_1$  and  $X_2$  be a random sample of size 2 from the uniform distribution on  $\{2, 4, 6, 8\}$ . Let  $Y = X_1 + X_2$ .

- (1) (10%) Find the **moment-generating function** of  $Y$ .
- (2) (5%) Find the **probability density function** (pdf) of  $Y$ .

4. (20%)  $X$  follows a normal distribution with mean  $\mu_x$  and variance 36. Based on  $n = 16$  observations and the corresponding sample mean  $\bar{x}$ , if we would like to test

$$H_0 : \mu_x = 50 \quad \text{versus} \quad H_1 : \mu_x > 50,$$

with the **critical region**:  $C = \{(x_1, x_2, \dots, x_{16}) : \bar{x} \geq 53\}$ .

- (1) (6%) What is the **size** of the test?
- (2) (8%) What is the  **$p$ -value** associated with  $\bar{x} = 54.5$ ?
- (3) (6%) What is the probability of **Type II error** when  $H_1 : \mu_x = 55$ ?

**(Remark:** Be careful, the sample size here is less than 30.)

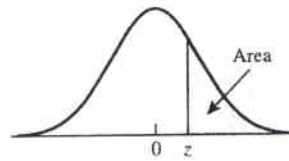
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備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。
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Table 4 Normal Curve Areas  
Standard normal probability in right-hand tail  
(for negative values of  $z$ , areas are found by symmetry)



Second decimal place of $z$										
$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.00135									
3.5	.000233									
4.0	.0000317									
4.5	.00000340									
5.0	.000000287									

From R. E. Walpole, *Introduction to Statistics* (New York: Macmillan, 1968).

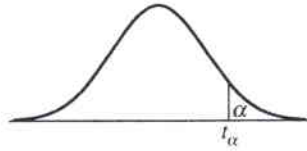
備

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- 二、試題請隨卷繳交。

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Table 5 Percentage Points of the *t* Distributions



$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	df
3.078	6.314	12.706	31.821	63.657	1
1.886	2.920	4.303	6.965	9.925	2
1.638	2.353	3.182	4.541	5.841	3
1.533	2.132	2.776	3.747	4.604	4
1.476	2.015	2.571	3.365	4.032	5
1.440	1.943	2.447	3.143	3.707	6
1.415	1.895	2.365	2.998	3.499	7
1.397	1.860	2.306	2.896	3.355	8
1.383	1.833	2.262	2.821	3.250	9
1.372	1.812	2.228	2.764	3.169	10
1.363	1.796	2.201	2.718	3.106	11
1.356	1.782	2.179	2.681	3.055	12
1.350	1.771	2.160	2.650	3.012	13
1.345	1.761	2.145	2.624	2.977	14
1.341	1.753	2.131	2.602	2.947	15
1.337	1.746	2.120	2.583	2.921	16
1.333	1.740	2.110	2.567	2.898	17
1.330	1.734	2.101	2.552	2.878	18
1.328	1.729	2.093	2.539	2.861	19
1.325	1.725	2.086	2.528	2.845	20
1.323	1.721	2.080	2.518	2.831	21
1.321	1.717	2.074	2.508	2.819	22
1.319	1.714	2.069	2.500	2.807	23
1.318	1.711	2.064	2.492	2.797	24
1.316	1.708	2.060	2.485	2.787	25
1.315	1.706	2.056	2.479	2.779	26
1.314	1.703	2.052	2.473	2.771	27
1.313	1.701	2.048	2.467	2.763	28
1.311	1.699	2.045	2.462	2.756	29
1.282	1.645	1.960	2.326	2.576	inf.

From "Table of Percentage Points of the *t*-Distribution." Computed by Maxine Merrington, *Biometrika*, Vol. 32 (1941), p. 300.

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考試科目	總體經濟學	系所別	經濟學系	考試時間	2 月 18 日(一) 第二節
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- Answer the following questions.
  - What are 'comovement' and 'recurrence' related to the business cycle facts? (8%)
  - How would R&D and technological innovations affect the steady-state values of capital-labor ratio, output per worker, and real GDP growth rate? (9%)
  - What is the central bank credibility? How does the use of inflation targeting improve central bank credibility? (8%)
- Analyze the effects on a large open economy of a temporary increase in tariff that hits both home and foreign country. Discuss the impact on the home country's national saving, investment, current account balance, and on the world real interest rate. (25%)
- The Tax Cuts and Jobs Act is the biggest gross tax cuts in American history, cutting over \$5.5 trillion in taxes over ten years. President Trump cut the corporate tax rate from 35 percent to 21 percent, the largest percentage point reduction of the top marginal rate in history. Moreover, President Trump's tax cuts include the biggest increase in the child tax credit in history. **Should the government** reduce taxes on capital? Are there any costs to such a policy? What would be the benefits? What would happen to real interest rates and real output and to investment and the capital stock? (25%)
- Suppose the monetary policy curve is given by  $r = 1 + 2\pi$ , and IS curve is given by  $Y = 4 - 2r$ , where  $r$  is the real interest rate,  $\pi$  is the inflation rate, and  $Y$  is the real output. Moreover, the Phillips curve is given by  $\pi = \pi^e + 0.5(Y - \bar{Y})$ , where  $\pi^e = 2\%$  is the expected inflation and  $\bar{Y} = 2$  is the full-employment output.
  - What is the short-run equilibrium values of aggregate output and inflation. (10%)
  - Assume now that the monetary policy curve is  $r = 1.25 + 2\pi$ . Does the new monetary policy curve represent an autonomous tightening or loosening of monetary policy? (5%)
  - What is the sacrifice ratio in this economy? Explain (10%)

備

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Please show all your work because (i) partial credit will sometimes be awarded and (ii) full credit may not be awarded for answers that appear without accompanying work.

1. (10 points each, Total 20 points) True or False. Please explain.
- (a) Since collusion yields higher profits for firms a one-time simultaneous quantity game, to collude and split the joint monopolist's payoff is the best strategy.
- (b) The compensated demand of a Giffen good is positively sloped.

2. (Total 30 points) Robin the Rueppell's fox (R) and Sarah the sand cat (S) live in an oasis in the Sahara Desert. Each is endowed with meat (x) and water (y). Robin has 5 kg of meat and 6 liters of water; Sarah has 3 kg of meat and 10 liters of water. Their preferences are

$$u_R(x_R, y_R) = x_R^{1/3} y_R^{2/3} \quad \text{and} \quad u_S(x_S, y_S) = \sqrt{x_S y_S}.$$

There is no other living creature in this oasis.

- (a) (2 points) What are their respective utilities of consuming their endowments?
- (b) (3 points) Can there be Pareto improvement? Why?
- (c) (15 points) If yes, how can they reach Pareto optimal through trade? [HINT: Let a liter of water be the numeraire and P be the price of meat per kg.]
- (d) (5 points) What is the equilibrium after the trade called? Who has the larger bargaining power?
- (e) (5 points) Draw an Edgeworth box to illustrate part (a) and part (b). Be sure to label everything clearly, including the origins, the endowment point, both Robin's and Sarah's indifference curves, etc..
3. (Total 30 points) The inverse demand for love potion is  $P(Q) = \alpha - \beta Q$ . Cupid is the monopolist of love potion and his cost function is  $Q^2 + \gamma Q$ .  $\alpha > \gamma > 0$ , and  $\beta > 2$ . Suppose he decides to use two-block pricing. Let  $Q_1$  be the quantity sold in the first block and  $Q_2$  be the total quantity sold.
- (a) (10 points) What are his optimal quantities sold in each block?
- (b) (15 points) Please draw a graph that depicts your answer in part (a) and the corresponding prices ( $P_1, P_2$ ). Also include the optimal uniform pricing ( $P^*$ ) and quantity  $Q^*$  on your graph as well. Your graph should include the demand, marginal revenue, cost, etc.. There is no need to calculate the uniform pricing, doing redundant calculation earn NO credit.
- (c) (5 points) From your graph, what is the relationship between ( $P_1, P_2$ ) and  $P^*$ , and the relationship between ( $Q_1, Q_2$ ) and  $Q^*$ ? What does this imply about the inefficiency of market power? [HINT: deadweight loss.]

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4. (Total 20 points) Barney has a continuous increasing and concave Bernoulli utility function  $u(\bullet)$  and initial wealth,  $w$ . There is a money tree that will grow \$100 with probability  $\pi$  and grow \$20 with probability  $1-\pi$ .
- (a) (5 points) If Barney owns the money tree, what is the minimum price he would sell it for?
- (b) (5 points) If Barney does not own the money tree, what is the maximum price he would buy it for?
- (c) (10 points) Are buying and selling prices equal? Please use ONE graph with money on the x-axis and utility on the y-axis to explain your answer. Given an economic intuition for your answer. [HINT: Certainty equivalent and risk premium.]



備

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