

3. Simulations

Six time series data with size 100 to 400 are generated from six models (3.1) to (3.6). A clear change period can be found in model (3.1) to (3.4). The figure of model(3.1) to (3.6) are also provided for a clear visual grasp of the data structure.

$$X_t = \begin{cases} t + \mathbf{e}_t & \mathbf{e}_t \sim N(0,1) & \text{if } 1 \leq t \leq 200 \\ 200 - t + \mathbf{e}_t, & \mathbf{e}_t \sim N(0,1) & \text{if } 201 \leq t \leq 400 \end{cases} \quad (3.1)$$

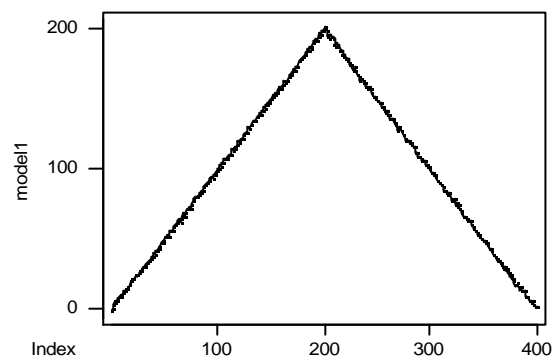
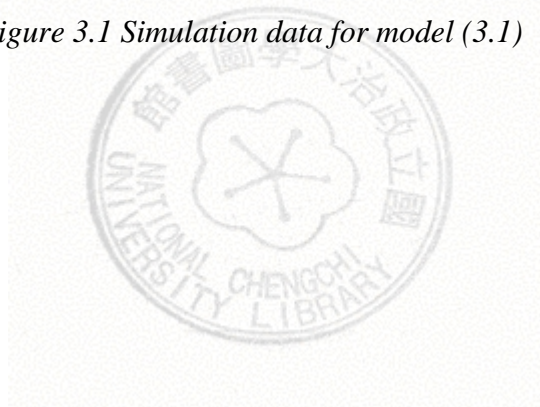


Figure 3.1 Simulation data for model (3.1)



$$X_t = \begin{cases} 200 - t + \mathbf{e}_t, & \mathbf{e}_t \sim N(0, 1) & \text{if } 1 \leq t \leq 200 \\ t + \mathbf{e}_t, & \mathbf{e}_t \sim N(0, 1) & \text{if } 201 \leq t \leq 400 \end{cases} \quad (3.2)$$

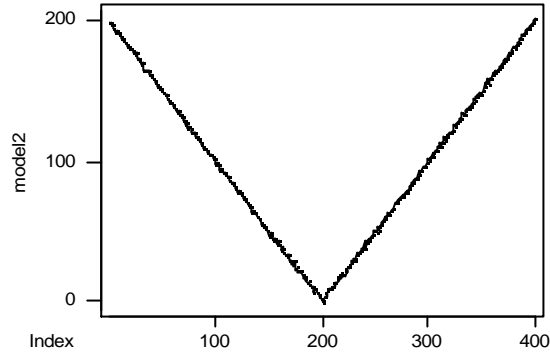


Figure 3.2 Simulation data for model (3.2)

$$X_t = \begin{cases} t^2 + t + \mathbf{e}_t, & \mathbf{e}_t \sim N(0, 1) & \text{if } 1 \leq t \leq 50 \\ 2550 - t^2 - t + \mathbf{e}_t, & \mathbf{e}_t \sim N(0, 1) & \text{if } 51 \leq t \leq 100 \end{cases} \quad (3.3)$$

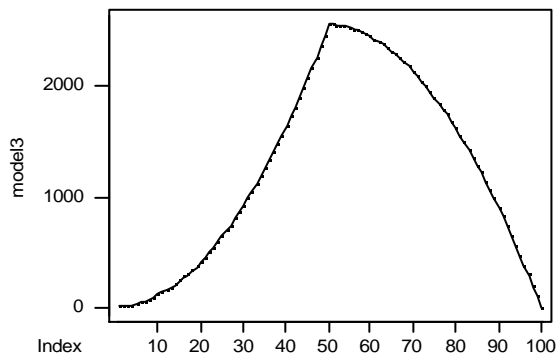


Figure (3.3) Simulation data for model (3.3)

$$X_t = \begin{cases} 2550 - t^2 - t + \mathbf{e}_t, & \mathbf{e}_t \sim N(0, 1) & \text{if } 1 \leq t \leq 50 \\ t^2 + t + \mathbf{e}_t, & \mathbf{e}_t \sim N(0, 1) & \text{if } 51 \leq t \leq 100 \end{cases} \quad (3.4)$$

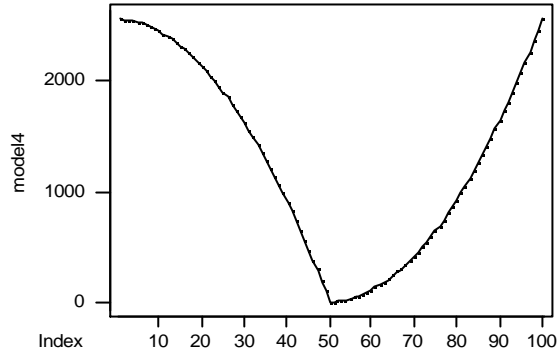


Figure (3.4) Simulation data for model (4.4)

$$X_t = \begin{cases} 0.2 + 1.5X_{t-1} - 0.5X_{t-2} + \mathbf{e}_t - 0.2\mathbf{e}_{t-1}, & \mathbf{e}_t \sim N(0, 1) & \text{if } 1 \leq t \leq 200 \\ 90 - 0.2 + 1.5X_{t-1} - 0.5X_{t-2} + \mathbf{e}_t - 0.2\mathbf{e}_{t-1}, & \mathbf{e}_t \sim N(0, 1) & \text{if } 201 \leq t \leq 400 \end{cases} \quad (3.5)$$

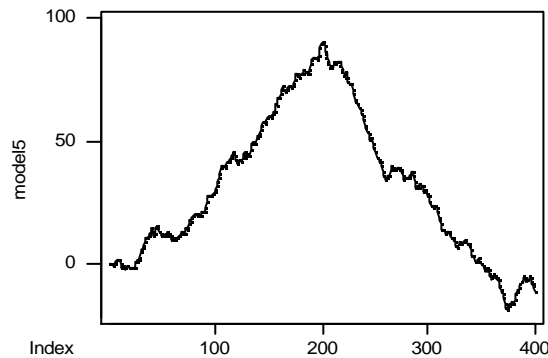


Figure (3.5) Simulation data for model (3.5)

$$X_t = \begin{cases} 14 - 0.2 + 1.5X_{t-1} - 0.5X_{t-2} + \mathbf{e}_t - 0.2\mathbf{e}_{t-1}, & \mathbf{e}_t \sim N(0,1) \text{ if } 1 \leq t \leq 200 \\ 0.2 + 1.5X_{t-1} - 0.5X_{t-2} + \mathbf{e}_t - 0.2\mathbf{e}_{t-1}, & \mathbf{e}_t \sim N(0,1) \text{ if } 201 \leq t \leq 400 \end{cases} \quad (3.6)$$

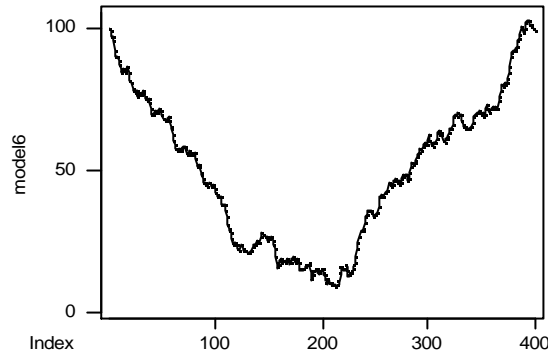


Figure (3.6) Simulation data for model (3.6)

In transforming the difference time series $\{Y_t\}$ into the fuzzy time series, we choose the linguistic values set: $U = \{L_i, i = 1, 2, 3, 4, 5\} = \{\text{fall sharply}, \text{fall}, \text{unchange}, \text{rise}, \text{rise sharply}\}$, and the membership function for model (3.1) to (3.6) are shown in Figure (3.7) to (3.12) respectively.

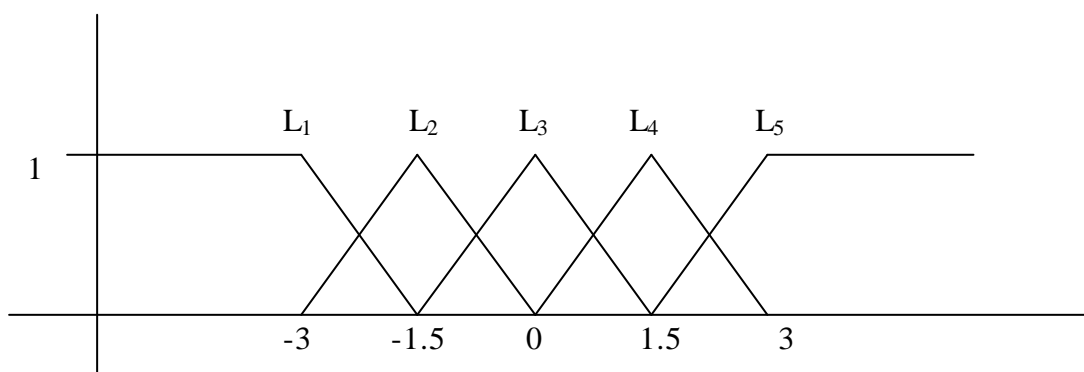


Figure (3.7) Membership function for model (3.1)

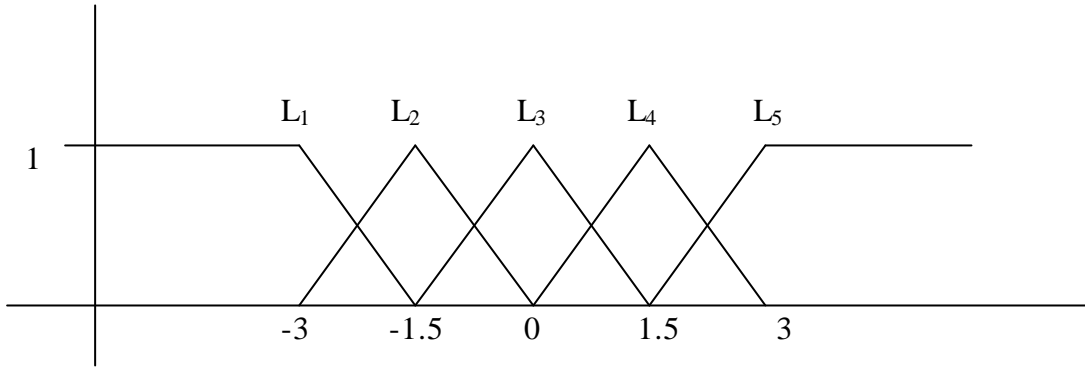


Figure (3.8) Membership function for model (3.2)

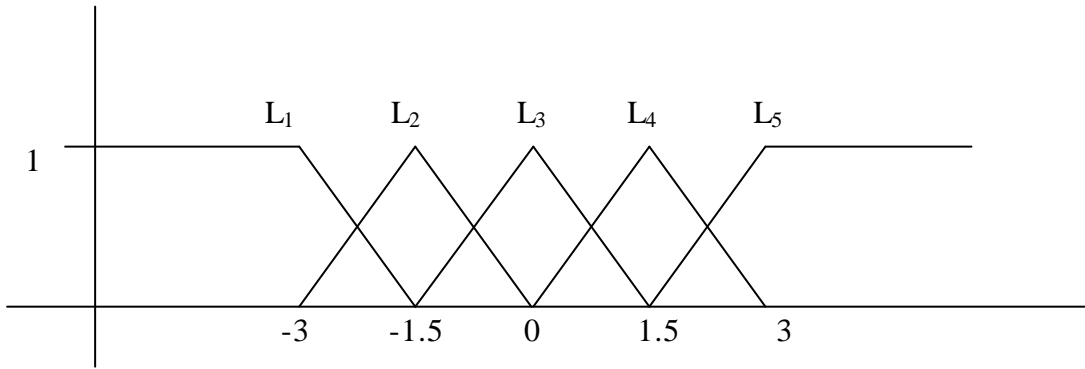


Figure (3.9) Membership function for model (3.3)

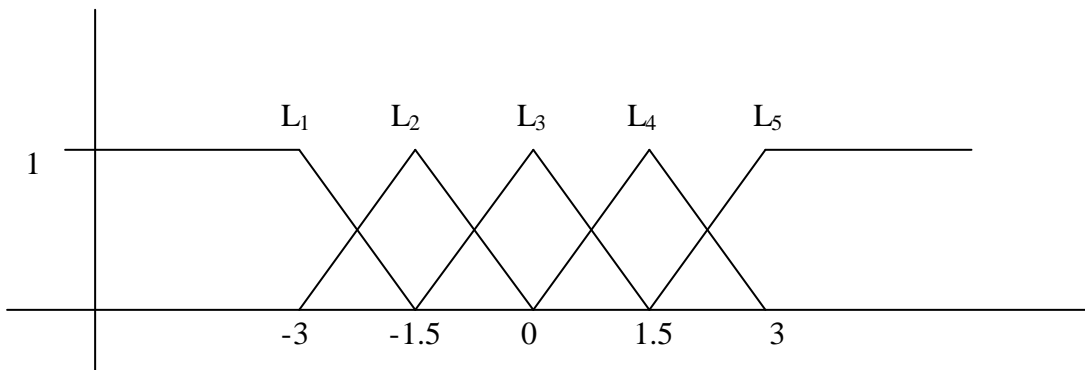


Figure (3.10) Membership function for model (3.4)

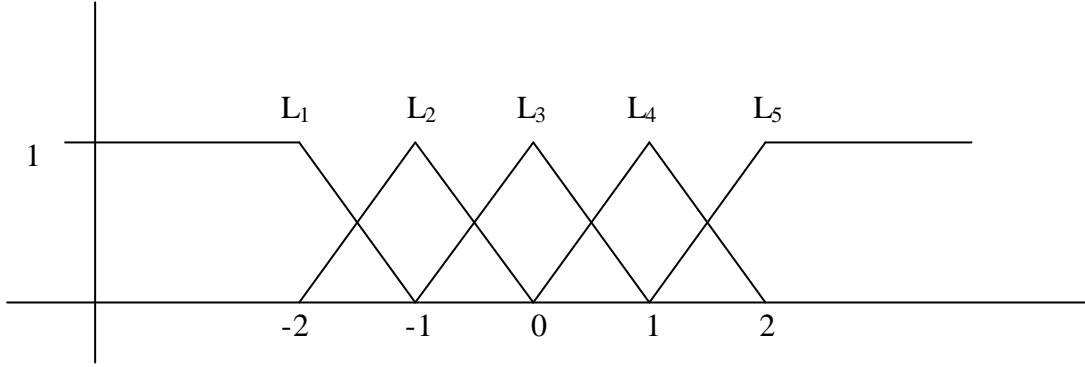


Figure (3.11) Membership function for model (3.5)

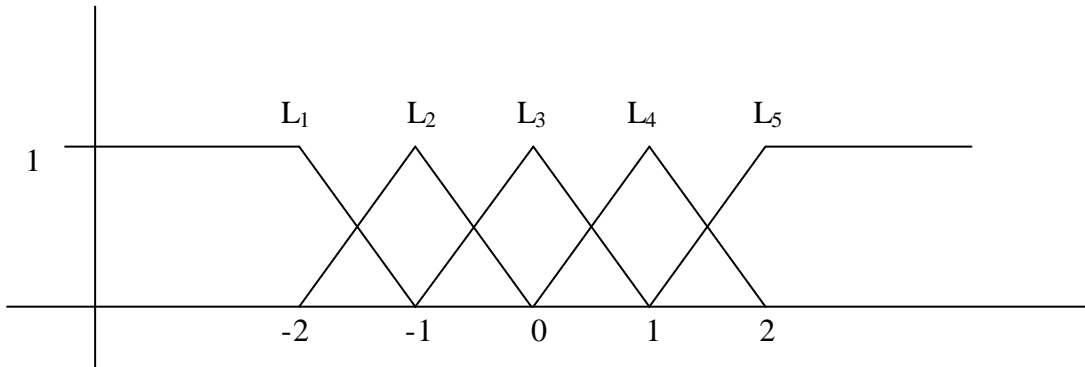


Figure (3.12) Membership function for model (3.6)

In model (3.1), we set the fuzzy weights a_1, a_2, a_3, a_4, a_5 to be, respectively, -2, -1, 0, 1, 2. The degree of the change period detection sequence and the h in definition 2.6 are set to be 10 and 6. It is obvious that there is a clear change at t_{200} and t_{201} where, in our setting, are the end of an upward trend and the beginning of a downward trend. The change periods we found are supposed to contain at least these two points. Following the process we proposed, we find a change period $T = \{t_{197}, t_{198}, \dots, t_{204}\}$ which contains t_{200} and t_{201} . We show the partial time series on T graphically in Figure (3.13), the twenty consecutive predecessors and successors are also included for a clearer visual grasp.

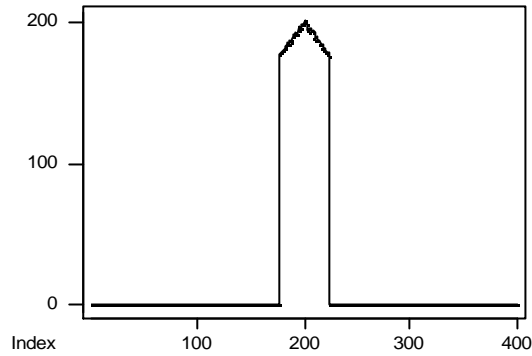


Figure (3.13) Partial time series at time 177 to 224 in model (4.1)

In model (3.2) to (3.6), the fuzzy weights a_1, a_2, a_3, a_4, a_5 , the degree of the change period detection sequence are set the same as that of model (3.1). A change period is detected at $T = \{t_{197}, t_{198}, \dots, t_{204}\}$ by letting $h = 6$. Figure (3.14) shows $X(177)$ to $X(224)$ of model (3.2).

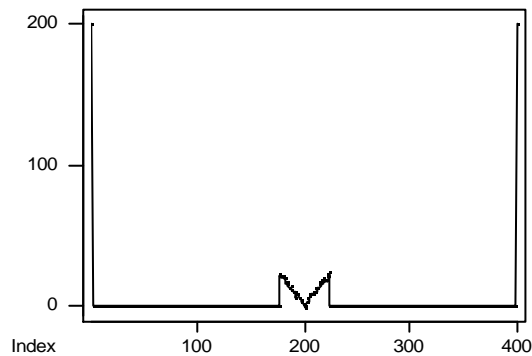


Figure (3.14) Partial time series at time 177 to 224 in model (3.2)

In model (3.3) and (3.4), we set $h = 15$, and we find a change period at $T = \{t_{47}, t_{48}, \dots, t_{51}\}$. We show the simulation results in Figure (3.15) and (3.16).

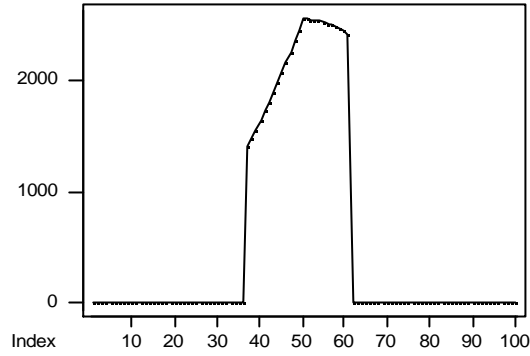


Figure (3.15) Partial time series at time 37 to 61 in model (3.3)

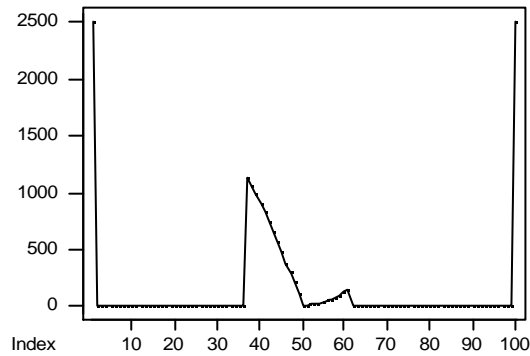


Figure (3.16) Partial time series at time 37 to 61 in model (3.4)

We can see from Figure (3.5) that there are many small change periods in model (3.5). You can decide whether or not to treat them as noises just by controlling the degree of the change period detection sequence. Let $h = 6$, and we have the change periods $T_1 = \{ t_6, t_7, \dots, t_{24} \}$, $T_2 = \{ t_{40}, t_{41}, \dots, t_{70} \}$, $T_3 = \{ t_{113}, t_{114}, \dots, t_{127} \}$, $T_4 = \{ t_{198}, t_{199}, \dots, t_{201} \}$, $T_5 = \{ t_{209}, t_{210}, \dots, t_{215} \}$, $T_6 = \{ t_{260}, t_{261}, \dots, t_{286} \}$, $T_7 = \{ t_{325}, t_{326}, \dots, t_{335} \}$, $T_8 = \{ t_{375}, t_{376}, t_{377} \}$ and $T_9 = \{ t_{391}, t_{392}, t_{393}, t_{394} \}$ in model (3.5).

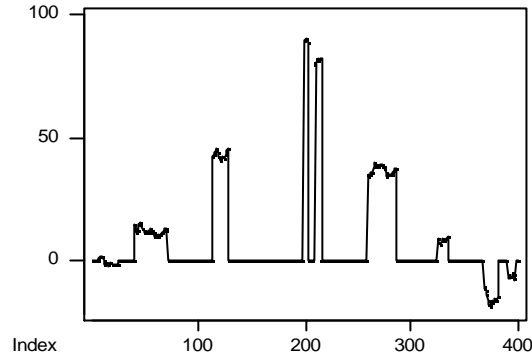


Figure (3.17) Partial time series at change periods of model (3.5)

Similarly, set $h = 6$ in model (3.6), we have the change periods $T_1 = \{t_{119}, t_{120}, \dots, t_{137}\}$, $T_2 = \{t_{140}, t_{141}, \dots, t_{149}\}$, $T_3 = \{t_{160}, t_{161}, \dots, t_{213}\}$, $T_4 = \{t_{220}, t_{221}, \dots, t_{228}\}$, $T_5 = \{t_{326}, t_{327}, \dots, t_{341}\}$.

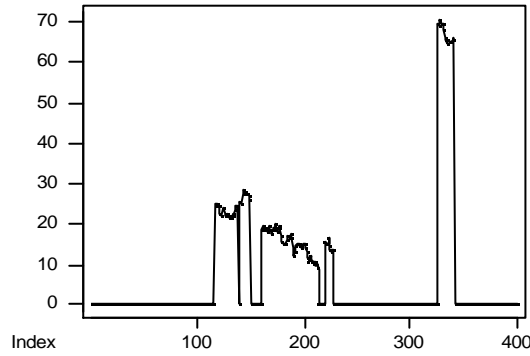


Figure (3.18) Partial time series at change periods of model (3.6)

We change the data structure at t_{200} and t_{201} in model (3.1), (3.2), (3.5), (3.6), and change the data structure at t_{50} and t_{51} in model (3.3) and (3.4). In each model, the change periods found by our approach accurately contain those points. As you can see in figure (3.17) and figure (3.18), we can even filter the noises and find the suitable change periods.