

Appendix

Derivation of Equation (3.12):

According to Equation (3.11), we have

$$V(0) = C_{0,n}(0) \cdot E_{Q^{0,n}} [\max(S_{0,n}(T_0) - K, 0) | F_0].$$

By Radon-Nikodym derivative, the probability measure can be changed from $Q^{0,n}$ to \hat{Q}^m

$$\begin{aligned} V(0) &= C_{0,n}(0) \cdot E_{\hat{Q}^m} \left[\frac{dQ^{0,n}}{d\hat{Q}^m} \max(S_{0,n}(T_0) - K, 0) | F_0 \right] \\ &= C_{0,n}(0) \cdot E_{\hat{Q}^m} \left[\frac{C_{0,n}(T_0)/C_{0,n}(0)}{\hat{C}_m(T_0)/\hat{C}_m(0)} \max(S_{0,n}(T_0) - K, 0) | F_0 \right] \end{aligned}$$

Replacing $C_{0,n}$ and C_m , we get

$V(0) =$

$$C_{0,n}(0) E_{\hat{Q}^m} \left[\frac{\frac{\sum_{i=1}^n (T_i - T_{i-1}) \cdot \bar{P}(T_0, T_i) \cdot Q(\tau > T_0 | F_{T_0})}{\sum_{i=1}^n (T_i - T_{i-1}) \cdot \bar{P}(0, T_i) \cdot Q(\tau > 0 | F_0)}}{\frac{(T_i - T_{i-1}) \cdot \bar{P}(T_0, T_m) \cdot Q(\tau > T_0 | F_{T_0})}{(T_i - T_{i-1}) \cdot \bar{P}(0, T_m) \cdot Q(\tau > 0 | F_0)}} \max(S_{0,n}(T_0) - K, 0) | F_0 \right],$$

Arranging the above formula, we thus can obtain

$$V(0) = \bar{P}(0, T_m) \cdot E_{\hat{Q}^m} \left[\frac{\sum_{i=1}^n (T_i - T_{i-1}) \cdot \bar{P}(T_0, T_i)}{\bar{P}(T_0, T_m)} \max(S_{0,n}(T_0) - K, 0) | F_0 \right].$$