

IV. Numerical Examples

Our numerical implementation focuses on Citizens First Bancorp Inc. The data sample covers from January 3, 2005 to May 29, 2007. We assumed interest rates to be deterministic, and use the zero curve stripped from the LIBOR rates and swap rate on May 29, 2007, as our main interest rate data. The recovery rate is assumed to be a constant at 40%, or loss given default equal to 60%.¹⁰ We simulate 50,000 paths to obtain CDS option values.

Table 1: CDS quotes for Citizens First Bancorp Inc on May 29, 2007.

CDS maturity (years)	Market quotes (basis points per year)
1	18.7
3	52.3
5	102.8
7	144
10	182

Table 1 shows the CDS quotes for Citizens First Bancorp Inc on May 29, 2007. We strip out the implied intensities based on the reduced form model, and Table 2 shows the implied intensities from the CDS quotes on May 29, 2007.¹¹ The survival

¹⁰ According to Hull and White (2000), hazard rate has little impact on the value of a CDS contracts.

¹¹ We assume the intensity function follows piece-wise constant.

probability at each time is presented in Figure 1.

Table 2: Implied intensities for Citizens First Bancorp Inc on May 29, 2007.

Time(years)	Intensities(%)
0~1	0.31
1~3	1.17
3~5	3.20
5~7	4.68
7~10	5.40

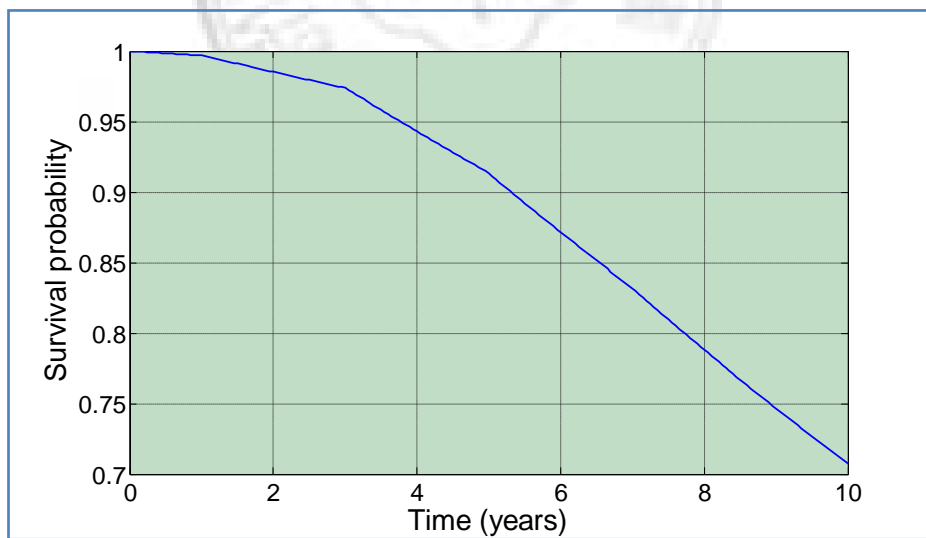


Figure 1: Survival probability curve for Citizens First Bancorp Inc.

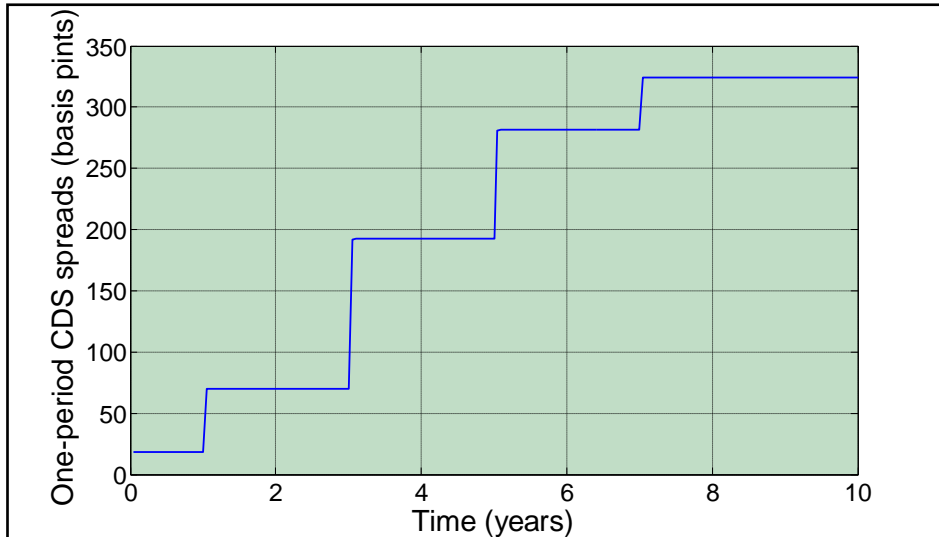


Figure 2: The one-period CDS spreads volatility structure for Citizens First Bancorp Inc on May 29, 2007.

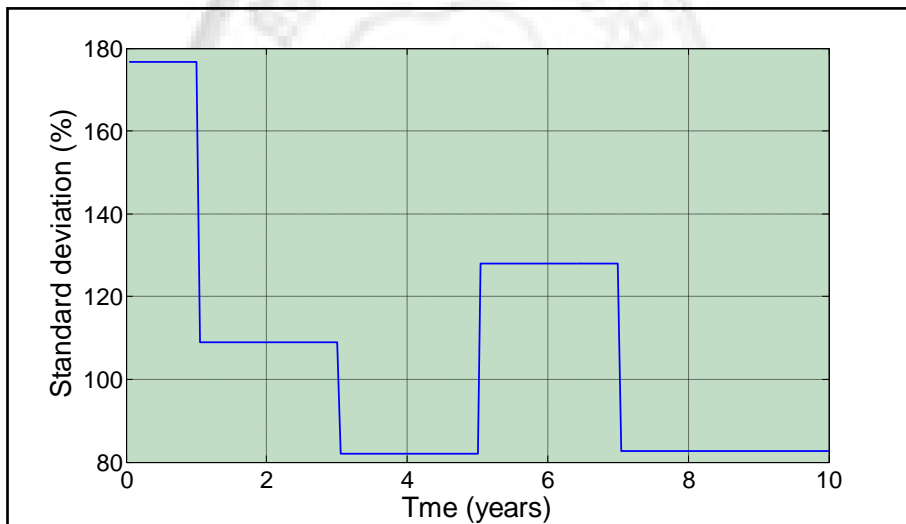


Figure 3: The volatility structure of one-period CDS spreads for Citizens First Bancorp Inc on May 29, 2007

We also split the 10-year time from May 29, 2007 into 200 small periods. The corresponding one-period CDS spreads are shown in Figure 2. The volatility for each one-period CDS spread is calculated from the historical data covering from January 3,

2005 to May 29, 2007, and the result is shown in Figure 3.

Table 3 and Table 4 show the LIBOR rates and the swap rate on May 29, 2007.

Based on these, the zero curve can be stripped out and the result is given in Figure 4.

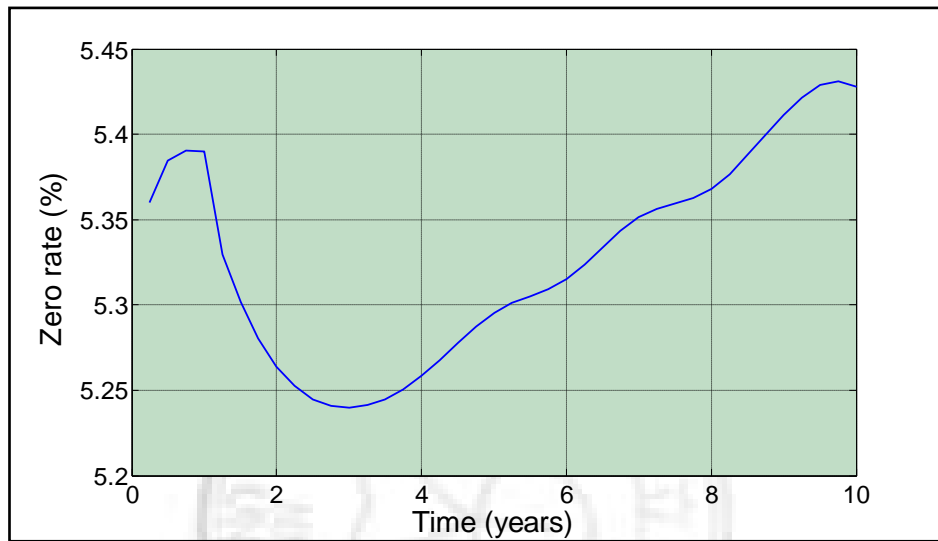


Figure 4: The zero curve on May 29, 2007

Table 3: LIBOR rates on May 29, 2007.

Time(years)	LIBOR rate(%)
0.25	5.36
0.5	5.3844
0.75	5.3904
1	5.39

Table 4: Swap rates on May 29, 2007.

Time(years)	Swap rate(%)
1	5.40
2	5.302
3	5.2775
4	5.294
5	5.3275
6	5.3455
7	5.377
8	5.392
9	5.428
10	5.4225

4.1. A comparison with the valuation models by Brigo (2004) and Hull and White (2002)

We first compare the values of European CDS options among different models. Figure 5 and Figure 6 present the European CDS options values at different exercise prices. The option maturity is 1 year for Figure 5 and 4 years for Figure 6, and both have a protection maturity of 5 years. The forward spread of the underlying CDS contract for the option maturing in 1year is 126.74 basis points, while the spread for the option expiring in 2 years is 192.27 basis points.

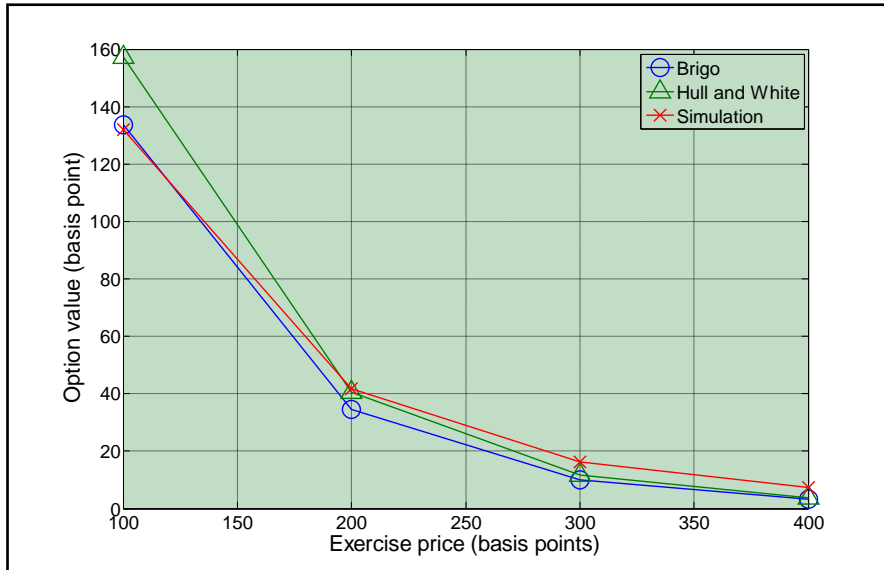


Figure 5: The comparison with the market formulas for European CDS options when the option maturity is 1 year.

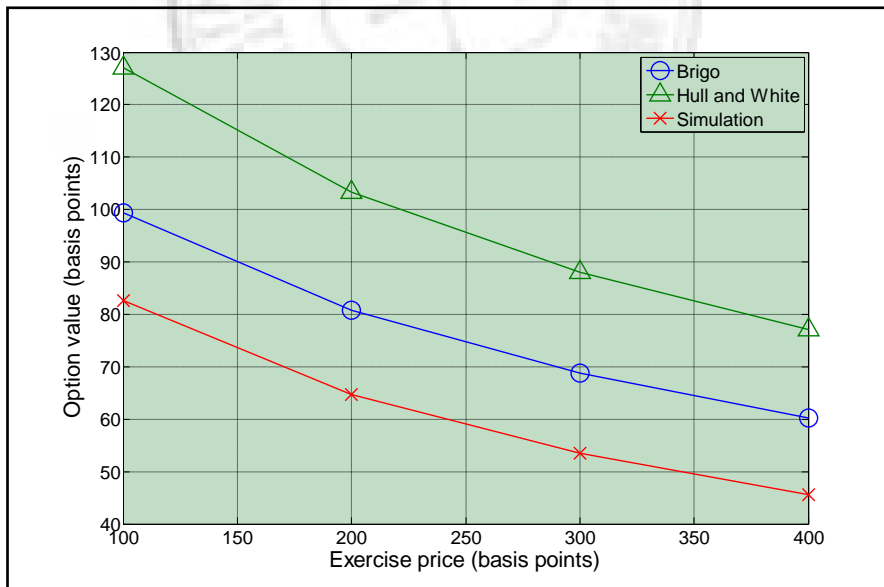


Figure 6: The comparison with the market formulas for European CDS options when the option maturity is 4 year.

In Figure 5, the option values from Hull and White's valuation model are close to those from Brigo's model and simulation results at higher exercise prices.¹² As the exercise is decreasing, the results from Hull and Whites exceed the other two. On the other hand, the results from simulation are all close to those from Brigo's model. The discrepancy between Hull and White's valuation model and the other two may be the latter models are based on the similar concepts.

Nevertheless, as the option maturity increases to 4 years, the simulation results begin to diverse from the Brigo's valuation model. The simulation produces lower values. The reason is that the Brigo's valuation model comes from the assumption that the CDS spread over a longer period follows a martingale. On the contrary, the dynamics of one-period spreads model is based on the idea that the one-period CDS spread which is of shorter length follows a martingale. When the option maturity is short, these two models provide almost the same results. As the option maturity gets longer, the distinction appears.

4.2. A comparison among European, Bermudan and American CDS options

The primary purpose of a Bermudan or American CDS option is to avoid the default by the reference company, which knocks out the option. During the option

¹² Instead of estimating default probabilities with default densities, we use default intensities to insert the market formula of European CDS option by Hull ad White (2002).

maturity, the holders of Bermudan or American CDS options can exercise their options once they realize the reference company is likely to default. In other words, the early-exercise feature provides the holders a type of insurance. This thought may be different from the intention of holding an American stock option.

Table5: Values of American, Bermudan and European CDS options for different exercise prices. The option maturity is 1 year and the protection maturity is 5 years.

Exercise(bps)	European(bps)	Bermudan(bps)	American(bps)
100	132.0308	132.5738	133.0748
	2.1455	1.6016	1.7572
200	41.6519	41.9842	42.6775
	1.9821	1.6469	1.7061
300	16.2501	16.3214	16.8703
	1.351	1.2815	1.1263
400	7.2573	7.2887	7.8567
	1.1864	1.1428	1.0126

Table 5 shows the values of European, Bermudan and American CDS options when the option maturity is 1 year and the protection maturity is 5 years. The smaller numbers represent the standard deviations. The Bermudan option is exercisable twice per year. As presented in Table 5, the values among the three types of options for different exercise prices do not differ from each other by more than 1 basis point. This probably implies that the market thinks that the default probability of Citizens First Bancorp Inc is not intense in one year. Consequently, the early-exercise premium does not appear significantly among the option values.

On the other hand, once the Bermudan or American CDS option is exercised, the holder starts to make payments at a fixed rate equal to exercise price for the underlying CDS contract. This means the holder starts to bear the volatility risk from the CDS spread. In other words, the market quote for the CDS contract probably reaches a level lower than the exercise price so that the originally cheaper payments become more expensive. Therefore the holder of a Bermudan or American CDS option becomes more reluctant to exercise before the option maturity. This may also be the reason why the values among European, Bermudan and American CDS options do not differ from each other under the situation that the default probability is not intense.

Table 6: Values of American, Bermudan and European CDS options for different exercise prices. The option maturity is 4 year and the protection maturity is 5 years.

Exercise(bps)	European(bps)	Bermudan(bps)	American(bps)
100	82.6411	193.5213	196.2631
	5.0992	2.852	3.8416
200	64.732	136.1216	138.7159
	4.8983	3.4901	3.3672
300	53.4808	105.5152	110.8493
	4.7052	3.7448	3.7111
400	45.5385	86.3139	90.6672
	4.5531	4.5732	4.768

From Table 6 in which the option maturity is 4 years, we can observe that the values of options with early-exercise features are much higher than those of the European CDS options. This reflects the default probability rises as the option

maturity gets longer. For the European CDS option, the risk of being knocked out increases. As a result, the European CDS option with longer option maturity produces a much lower value.

Furthermore, compared with the in-the-money value of the European CDS option with the option maturity 1 year, the European CDS option with the option maturity 4 years produces a lower value. This means that the holder of the European CDS option bears most risk of default as the option is deep-in-the money. Consequently, the value drops as the option maturity gets longer.

Table 7 through 9 shows the option values with different protection maturities and same option maturities. As shown in these tables, the option value with longer protection maturity has greater value. This is because the option with longer protection maturity provides longer term of the protection against the risk of default by the reference company when it is exercised. Therefore the option with longer protection maturity is more valuable.

Table 7: Values of American, Bermudan and European CDS options for different exercise prices. The option maturity is 1 year and the protection maturity is 4 years.

Exercise(bps)	European(bps)	Bermudan(bps)	American(bps)
100	102.8262 1.671	102.8262 1.671	104.105 0.8812
200	32.4387 1.5436	32.4387 1.5436	32.771 0.9112
300	12.6557 1.0521	12.6557 1.0521	13.285 0.8871
400	5.652 0.924	5.652 0.924	6.1535 0.7915

Table 8: Values of American, Bermudan and European CDS options for different exercise prices. The option maturity is 1 year and the protection maturity is 3 years.

Exercise(bps)	European(bps)	Bermudan(bps)	American(bps)
100	35.7314 1.1428	36.0838 1.1708	36.6902 0.8877
200	16.2895 0.9819	16.6291 0.9185	16.8651 0.6222
300	8.8035 0.7374	9.0122 0.7297	9.3004 0.5623
400	5.3227 0.6029	5.4738 0.5846	5.8842 0.3936

Table 9: Values of American, Bermudan and European CDS options for different exercise prices. The option maturity is 1 year and the protection maturity is 2 years.

Exercise(bps)	European(bps)	Bermudan(bps)	American(bps)
100	19.6426 0.7196	19.8088 0.5762	20.0759 0.3979
200	9.3087 0.7113	9.3654 0.68	9.7437 0.5403
300	5.211 0.6952	5.2455 0.7008	5.5177 0.625
400	3.2194 0.6295	3.2464 0.616	3.4413 0.6824

4.3. Sensitivity analysis-changes in market quotes for CDS contracts

Let us further look at the how option values change with the spreads. We test the sensitivity of option values to CDS quotes in this section, which is implemented by observing how the option values alter with 10% or -10% changes of CDS quotes. Table 10 presents the spreads when market quotes are increased or decreased by 10%. According to this table, Figure 7 shows the resulting one-period CDS spread structure, and Table 11 and Table 12 show the corresponding option values when the option maturity is 1 year and the protection maturity is 5 years. The numbers in the bracket mean the percentage change from the original option values.

Table 10: The market quotes with a +10% or -10% change.

Time (years)	CDS quotes with 10% change (bps)	Original CDS quotes (bps)	CDS quotes with -10% change (bps)
1	20.57	18.7	16.83
3	57.53	52.3	47.07
5	113.08	102.8	92.52
7	158.4	144	129.6
10	200.2	182	163.8

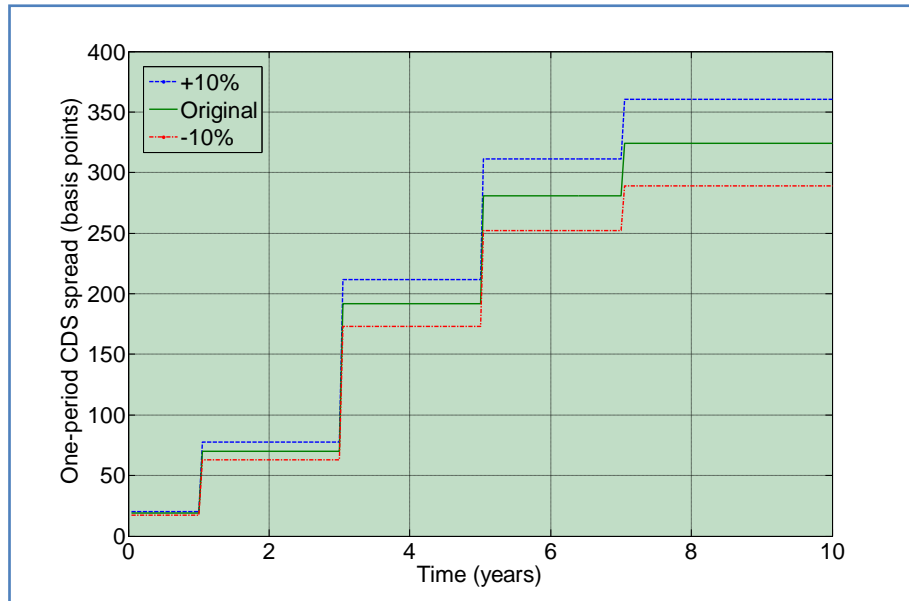


Figure 7: The one-period CDS spreads structure with +10% or -10% change.

Table 11: Values of American, Bermudan and European CDS options for different exercise prices when the CDS quotes are 10% higher. The option maturity is 1 year and the protection maturity is 5 years.

Exercise(bps)	European(bps)	Bermudan(bps)	American(bps)
100	161.5074 (22.33%) 2.2524	162.0869 (22.26%) 1.3931	162.4584 (22.08%) 1.9058
200	54.9404 (31.90%) 2.2649	55.2149 (31.51%) 1.7793	55.5642 (30.20%) 1.3035
300	22.3555 (37.57%) 1.6183	22.5107 (37.92%) 1.4764	23.4532 (39.02%) 1.5321
400	10.3065 (42.02%) 1.3437	10.3327 (41.76%) 1.2694	11.1791 (42.29%) 1.1439

Table 12: Values of American, Bermudan and European CDS options for different exercise prices when the CDS quotes are 10% lower. The option maturity is 1 year and the protection maturity is 5 years.

Exercise(bps)	European(bps)	Bermudan(bps)	American(bps)
100	102.9233 (-22.05%) 3.8883	103.1157 (-22.22%) 3.8377	103.2561 (-22.41%) 4.0109
200	29.9753 (-28.03%) 2.427	30.015 (-28.51%) 2.4593	30.5084 (-28.51%) 2.7001
300	11.4501 (-29.54%) 1.3303	11.4901 (-29.60%) 1.3637	11.8323 (-29.86%) 1.5028
400	5.2883 (-27.13%) 0.9111	5.3443 (-26.68%) 0.9389	5.6874 (-27.61%) 0.8005

Obviously, the option values are sensitive to the change of the CDS quotes. Table 11 and 12 appear that the percentage changes of the option values are more than the absolute value of 10%. This implies that the option values are sensitive to the CDS quotes. In addition, as the option is more out-of-the-money, the respective value is more sensitive to the CDS quotes. This is consistent with that of stock options. On the other hand, when the option maturity is increased to 4 years, the sensitivities of the option values to the CDS quotes diminish.

Table 13: Values of American, Bermudan and European CDS options for different exercise prices when the CDS quotes are 10% higher. The option maturity is 4 year and the protection maturity is 5 years.

Exercise(bps)	European(bps)	Bermudan(bps)	American(bps)
100	92.0802 (11.42%) 5.422	221.2105 (14.31%) 4.1159	224.7272 (14.50%) 3.7655
200	72.8288 (12.51%) 5.226	155.681 (14.37%) 3.7696	158.7479 (14.44%) 3.3583
300	60.5612 (13.24%) 5.0348	122.4255 (16.03%) 4.3824	127.1713 (14.72%) 4.6901
400	51.8073 (13.77%) 4.8598	100.0265 (15.89%) 4.9481	105.3398 (16.18%) 5.1272

Table 14: Values of American, Bermudan and European CDS options for different exercise prices when the CDS quotes are 10% lower. The option maturity is 4 year and the protection maturity is 5 years.

Exercise(bps)	European(bps)	Bermudan(bps)	American(bps)
100	74.3411 (-10.04%) 4.1421	167.7756 (-11.85%) 5.0878	170.59 (-14.51%) 5.8989
200	57.5824 (-11.04%) 4.0127	116.7299 (-11.80%) 3.6608	120.0561 (-15.85%) 4.4944
300	47.2341 (-11.68%) 3.9258	90.0379 (-10.90%) 2.9905	94.0193 (-18.77%) 3.272
400	40.0337 (-12.09%) 3.8793	72.9746 (-10.51%) 2.243	77.2434 (-19.51%) 3.9913

4.4. Sensitivity analysis-changes in volatilities of one-period CDS spreads

Figure 8 presents the volatility structure for one-period CDS spreads when standard deviation is increased or decreased by 10%. Table 15 and 16 show the option values according to these changes when the option maturity is 1 year and the protection maturity is 5 years.

Table 15: Values of American, Bermudan and European CDS options for different exercise prices when the volatilities of one-period CDS spreads are 10% higher
The option maturity is 1 year and the protection maturity is 5 years.

Exercise(bps)	European(bps)	Bermudan(bps)	American(bps)
100	137.2682	138.3709	138.8918
	(3.97%)	(4.77%)	(3.98%)
	5.4341	4.4301	4.327
200	50.8453	51.2517	52.328
	(22.07%)	(24.64%)	(20.09%)
	4.1743	3.9947	3.7375
300	23.2257	23.5027	24.9035
	(42.93%)	(52.58%)	(39.31%)
	2.7984	2.9079	2.7929
400	12.1639	12.3396	13.3146
	(67.61%)	(82.67%)	(57.06%)
	1.9289	1.969	2.0998

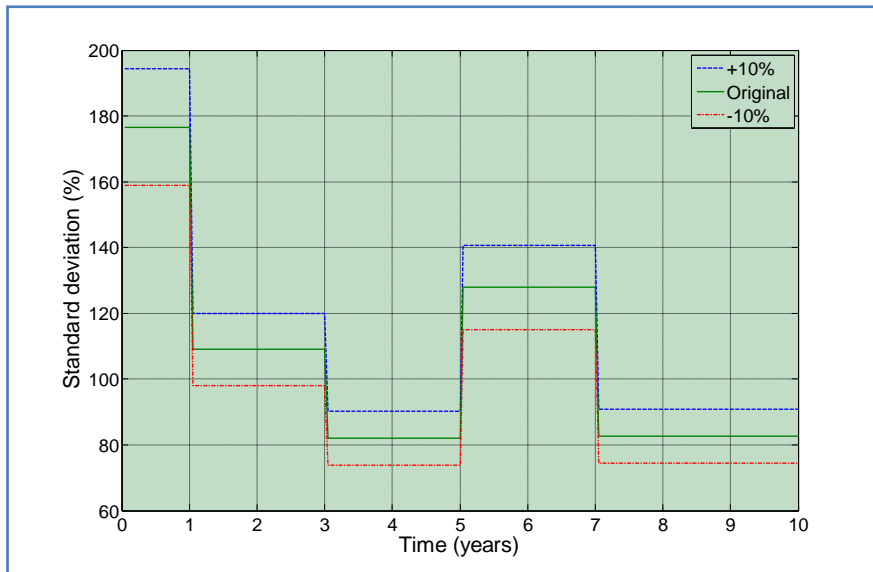


Figure 8: The volatility structure for one-period CDS spreads with +10% or -10% change.

Table 16: Values of American, Bermudan and European CDS options for different exercise prices when the volatilities of one-period CDS spreads are 10% lower. The option maturity is 1 year and the protection maturity is 5 years.

Exercise(bps)	European(bps)	Bermudan(bps)	American(bps)
100	124.2696 (-5.88%) 1.5602	124.3952 (-6.17%) 1.5603	124.7268 (-6.27%) 2.2983
200	32.6904 (-21.52%) 0.7062	32.7648 (-21.96%) 0.6343	32.9401 (-22.82%) 0.7423
300	10.9748 (-32.46%) 0.6522	10.9719 (-32.78%) 0.6478	11.2628 (-33.24%) 0.5175
400	4.4267 (-39.00%) 0.5668	4.4394 (-39.09%) 0.5687	4.6924 (-40.28%) 0.6011

As Table 15 and Table 16 show, the option values are insensitive to the change of the volatility structure when the options are in the money and are sensitive when the options are out of money. The reason that the out-of-money options are sensitive may be that higher volatilities raise the probability that the option will be in the money in the future. As a result, the option values dramatically increase as the volatilities of one-period CDS spreads rise.

Table 17 and Table 18 present the option values with option maturity 4 years when the standard deviations of one-period CDS spreads are increased or decreased by 10%. The option values are less sensitive to the volatilities of one-period spreads when the option maturities are longer. It is interesting to note that the increased volatilities may have negative effects on the European CDS options. As shown in Table 17, the option values at exercise prices 100 and 200 bps slightly decrease when the volatilities of one-period CDS spreads rise.

Table 17: Values of American, Bermudan and European CDS options for different exercise prices when the volatilities of one-period CDS spreads are 10% higher
The option maturity is 4 year and the protection maturity is 5 years.

Exercise(bps)	European(bps)	Bermudan(bps)	American(bps)
100	80.4562 (-2.64%) 4.5528	202.6163 (4.70%) 6.3941	206.524 (5.23%) 6.0447
200	64.6484 (-0.13%) 4.4024	146.582 (7.68%) 6.0864	150 (8.13%) 6.0629
300	54.5181 (1.94%) 4.3713	115.24 (9.22%) 4.8146	121.7752 (9.86%) 5.5289
400	47.3005 (3.87%) 4.4046	95.3466 (10.46%) 3.0207	101.9943 (12.49%) 3.5922

Table 18: Values of American, Bermudan and European CDS options for different exercise prices when the volatilities of one-period CDS spreads are 10% lower
The option maturity is 4 year and the protection maturity is 5 years.

Exercise(bps)	European(bps)	Bermudan(bps)	American(bps)
100	81.8166 (-1.00%) 1.7234	184.9958 (-4.37%) 1.2317	185.0731 (-5.74%) 2.2325
200	61.3957 (-5.15%) 1.6276	127.7967 (-4.95%) 4.0074	129.3831 (-7.87%) 3.5778
300	48.6807 (-8.98%) 1.4349	98.2298 (-3.76%) 5.2217	101.5458 (-11.38%) 4.7527
400	39.8813 (-12.42%) 1.2809	78.0117 (-7.15%) 5.1995	80.1463 (-13.96%) 5.0865