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A Panel Stationarity Test with Cross Section Dependence

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Abstract

Many less attentions are devoted to the development of testing for stationarity null in the panel context, despite that the testing framework has been proven to be useful and natural in many empirical panel studies. More than this, recent tests for panel unit root suggested are constructed in the presence of cross section dependence that has been thought of as an important practical reality in real data. The project proposes a test for panel stationarity null in the presence of cross section dependence, in contrast to the development in the panel unit root testing. Useful information to explain the variations of a particular series is contained in those of the rest series as covariates due to the presence of cross section dependence. The presence of cross section dependence is thus a merit to have in terms of power advantages. Our Monte Carlo evidence consistently demonstrates that the power gains for the tests increase, as the correlation between the covariates and the individual series increases. Also, the suggested test shares the optimality property as its univariate counterpart that has the most power against the local simple alternative.

JEL Classification Codes: C12, C15, C22
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1 Introduction and Summary

The problem of testing for stationarity in heterogeneous panels has not attracted as much attention as it deserves, as that for testing for unit roots in the literature. To name just a few, Choi (2001), Im, Pesaran and Shin (2003), Levin, Lin and Chu (2002), and Maddala and Wu (1999) consist of early important contributions to testing for unit roots using panel data. These proposed tests however are all constructed on the assumption of cross section independence. The assumption is not deemed to be quite realistic when applying the tests to the context of cross country regression where the cases with correlations among countries are found to be general, rather exceptional. Failing to control for cross section dependence could result in substantial size distortions for these panel unit root tests, as many simulations have documented. In a response to the unsatisfactory treatments in the early literature, the recent development of new panel unit root tests focuses on how to appropriately account for the cross section dependence. For example, Chang (2002), Phillips and Sul (2003), and Moon and Perron (2003) exploit orthogonalization-type procedures to eliminate the cross section dependence asymtotically, while Pesaran (2003) considers the Dicky-Fuller type regression that is augmented the cross section averages of lagged levels and first-differences of the individual series. However, it is clear that there are not many parallel developments for panel testing for stationarity. Even up to now, to our knowledge, Hadri (2000) remains the only one that considers testing for stationarity in the panel context. The tests suggested are built on the restrictive assumption of cross section independence, and thus its applicability to the real data may well be limited. Yet, no available tests for stationarity have been developed that is capable of correcting for cross section dependence in the context of heterogeneous panels.

One of potential reasons for the slow development of testing for panel stationaity might well be that professionals have been accustomed to the use of panel unit root tests. Nevertheless whether to apply any existing panel unit root test depends on the nature of the questions under investigation, and implications emerging from testing exercises using panel unit root tests might vary with the testing results. For the aforementioned studies on the panel unit root testing, as an illustration, researchers frequently look at testing the purchasing power parity hypothesis by applying the suggested tests to panels of real exchange rates, typically, from OECD countries. Testing results more often than not point to rejections to the null of unit roots for the real exchange rate panel under study, having been interpreted as evidence in favor of PPP hypothesis. However, rejections by the sort of testing procedures can at best indicate that some of real exchange rate series in the panel do not display long-run stochastic trending behavior. A conclusion that PPP holds in the long run might well be premature as well as misleading. When inquiries into whether PPP stands well in panel data is of concern, it may be more appealing to investigate the issue in the framework of testing for the stationarity null. This fits with the classical statistical testing spirit where the null hypothesis should reflect the belief by researchers. Most of economists tend to believe that PPP may hold at least in the long run, implying the deviations from the long run equilibrium of real exchange rate would exhibit stationarity after all. Testing for PPP using the cross country regression, therefore, amounts to a problem of testing for the stationarity null in the panel of real exchange rate series. Unless there is strong evidence presented against the null of stationarity, researchers will not reject lightly the null hypothesis. With panel data more available now, this immediately calls for a need to develop tests for stationarity in panels.

The development of testing for the stationarity null was indeed not left far behind for the univariate case. The seminal work of Kwiatkowski, Phillips, Schmidt, and Shin (1992, hereafter KPSS for shirt) first propose tests for the stationarity null in an error component model. The tests share an optimality property that is most powerful against simple local alternatives. Some modifications to the tests also have been proposed in the literature to correct for the size problems associated with the tests. Many empirical studies employ KPSS tests as a complementary check on the time series property for the data after looking into unit root tests. On the other hand, following construction of panel unit root tests, Hadri (2000) mounts panel tests for stationarity null by taking average over individual KPSS test statistics, on the restrictive independence assumption. Not much further progress along the line has been ever made since.

There should exist some technical difficulties when considering tests for stationarity in panels in the presence of cross section dependence. First, when building tests for panel stationarity null on the error component model, it is to be examined whether either the orthogonalization technique adopted by Phillips and Sul (2003) and Moon and Perron (2003), or the augmentation method with cross section average by Pesaran (2003) to remove cross section dependence can be applied equally well in the context. It should be emphasized that none of the available panel unit root tests is built on the error component framework. More importantly, whether or not the proposed tests for panel stationarity could continue enjoying the power optimality property of the univariate KPSS tests should be the focal point when constructing the sort of the tests.

This project attempts to answer part of the above questions of concern, in particular the power optimality of the panel stationarity tests. The surge in the construction of panel unit root tests was initially triggered by the need to improve over the univariate unit root tests that are found to lack power. Quite similar findings for low discrimination ability for KPSS tests has been established. Pooling stionarity tests over cross sections to increase power, comparable to the panel unit root testing, thus constitutes a natural direction to move in the literature. One of potential contributions about the project lies in suggesting a test for panel stationarity null in the presence of cross section dependence, while maintaining a power optimality.

While it seems to be a difficult notion to recognize, the presence of cross section dependence in the data in fact is not as bad as what it was thought of. Instead, the power of the proposed tests can be enhanced more in the cases with cross section dependence. The key to understand this is that the presence of cross section dependence offers any series under test candidates of covariate variables. In the regression context, we note that adding regressors of explanatory power reduces the estimation risk of regression disturbances. The fact is made use of and embodied into the construction of the proposed tests. Specifically, for each individual series, we add into each regression the corresponding covariates before computing the individual test for stationarity. Because the covariates are due to the presence of cross section dependence, they posses good explanatory power, as a result increasing the signal-to-noise ratio in the regression, and also the power for each individual test. Generally our simulations support the idea where the power gains for each individual tests is increased, as the correlation between the covariates and the individual series increases. Thus, the information contained in the covariates due to the presence of cross section dependence is extremely useful. Testing results using the idea of covariates can be first found in Hansen (1995). Elliot and Jansson (2002) extend the idea to the context of efficient unit root testing. While the idea is simple to catch, applied researchers complains about the choice of covariate variables when coming to the implementation of the idea. With the existence of cross sections dependence, the covariates with each series comes naturally to be the rest series in the panel. The difficulty with the choice of covariates vanishes in the panel testing in the presence of cross section dependence. Distinct from the aforementioned tests, our proposed tests are derived in the panel context and focus on testing for stationarity null. The suggested tests are derived using the Neyman-Pearson lemma, and thus possess the power optimality as the univariate counterpart does.

The rest of the report is organized as follows. Section 2 describes a simple panel with cross section dependence as the DGP. Section 3 derives the test statistics in the presence of cross section dependence. The power gain of the test proposed due to the presence of cross section dependence is also briefly discussed. Section 4 then gives an self-evaluation of the whole project.

2 A Simple Dynamic Panel with Cross Section Dependence

Let y_{it} be the observation on the ith cross-section unit at time t that is generated by the following simple unobservable component model:

$$y_{it} = \mu_{it} + u_{it}, \quad i = 1, \cdots, N; \ t = 1, \cdots, T$$

where the deterministic part of the model

$$\mu_{it} = \sum_{j=1}^{p} \beta_{ij} t^j$$

and the error term u_{it} has the one-factor structure

$$u_{it} = \gamma_i f_t + \varepsilon_{it},$$

It should be noted that f_t is understood to be the unobserved common effect, while ε_{it} is the individual specific error. The one-factor error structure gives birth to the presence of cross section dependence where the covariance matrix for the errors at time t can be illustrated as:

$$\Omega = \sigma_f^2 \begin{pmatrix} \gamma_1^2 & \gamma_1\gamma_2 & \cdots & \gamma_1\gamma_N \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \gamma_N\gamma_1 & \gamma_N\gamma_2 & \cdots & \gamma_N^2 \end{pmatrix} + \sigma_\varepsilon^2 I_N,$$

The following assumptions are needed for the asymptotic derivation of the proposed test statistics. A(1): The individual shocks ε_{it} are independently distributed both over time and cross-section, have mean zero, finite variance σ_ε^2, and finite fourth-oder moment. A(2): The common effect f_t is serially uncorrelated with mean zero and a constant variance σ_f^2, and finite fourth-order moment. A(3): $\varepsilon_{it}, \ f_t, \ \gamma_i$ are independently distributed for all i. A(1) and A(2) jointly suggest that the error u_{it} is serially uncorrelated. But in general it is easy to introduce serial correlation into the process by letting $u_{it} = \sum_{j=1}^{q} \rho_{ij} u_{i,t-1} + \gamma_i f_i + \varepsilon_{it}$. Alternative non-parametric procedures to bring into serial correlation can be considered as well. Throughout, to ease the exposition, we shall derive the test statistics under the simple case where the error is serially uncorrelated.

3 Panel Testing for Stationarity with Cross Section Dependence

The permanent-transitory decomposition for y_t can be written as

$$y_t = \mu_t + (1-\theta)\sum_{s=1}^{t-1} u_s + u_t,$$

where

$$\begin{aligned}
y_t &= (y_{1t}, \cdots, y_{Nt})', \\
\mu_t &= \left(\sum_{j=0}^{p} \beta_{1j} t^j, \sum_{j=0}^{p} \beta_{2j} t^j, \cdots, \sum_{j=0}^{p} \beta_{Nj} t^j\right)', \\
\theta &= (\theta_1, \theta_2, \cdots, \theta_N)', \\
u_t &= (\gamma_1 f_t + \varepsilon_{1t}, \gamma_2 f_t + \varepsilon_{2t}, \cdots, \gamma_N f_t + \varepsilon_{Nt})'.
\end{aligned}$$

We are interested in the testing problem:

$$H_0 : \theta_i = 1 \quad \text{for all } i, \text{ vs. } H_1 : |\theta_i| < 1, \quad i = 1, 2, \cdots, N_1; \theta_i = 1, \quad i = N_1 + 1, N_1 + 2, \cdots, N.$$

To see how the presence of cross section dependence brings forth the power gains of the testing problem, consider an individual transformed process who has the the following decomposition:

$$y_{it} - \omega_{i\cdot}\Omega_{\cdot\cdot}^{-1} y_{\cdot t} = \mu_{it}^{i\cdot} + (1-\theta_i)\sum_{s=1}^{t-1} u_{is} + u_{it}^{i\cdot},$$

where

$$\begin{aligned}
\mu_{it}^{i\cdot} &= \mu_{it} - \omega_{i\cdot}\Omega_{\cdot\cdot}^{-1}\mu_{\cdot t}, \\
u_{it}^{i\cdot} &= u_{it} - \omega_{i\cdot}\Omega_{\cdot\cdot}^{-1} u_{\cdot t}, \\
\rho_i^2 &= \omega_{ii}^{-1}\sigma_{i\cdot}\Omega_{\cdot\cdot}^{-1}\sigma_{\cdot i}.
\end{aligned}$$

where any variable such as $x_{i.}$ denotes the vector or matrix who is made up of ith row and all the columns in x except i, and $x_{..}$ consisting of all the columns and rows in x except ith column and ith row. Now note that $Var(u_{it}^{i\cdot}) = (1 - \rho_i Var(u_{it})$. Therefore, the transformation decreases the variance of the transitory component by a proportion ρ^2, an increase in the signal-to-noise ratio in the transformed series. The covariates $y_{\cdot t}$ due to the presence of cross section dependence brings about a power gains to the testing problem. And the power gains could be quite substantial if the degree of cross section dependence is high, summarized by the parameter ρ_i^2.

Now we turn to establish the test statistics in the presence of cross section dependence. Let

$$
\begin{aligned}
&C_{it} = (1, \cdots, t^p)' \quad \text{for all } i,\\
&C_t = I_N \otimes C_{it},\\
&\beta = (\beta_{10}, \cdots, \beta_{1p}, \beta_{20}, \cdots, \beta_{2p}, \cdots, \beta_{N0}, \cdots, \beta_{Np}),
\end{aligned}
$$

So with the notations, the simple dynamic panel can be re-written as

$$
y_t = C_t'\beta + u_t.
$$

The testing problem is invariant under the group transformation $y_t \rightarrow \; y_t + C_t'b, \quad b \in R^{N(p+1)}$. The probability density of the maximal invariant, given that Ω is known, is proportion to $\exp\left(-\frac{1}{2}\sum_{t=1}^{T}\tilde{u}_t(\theta;\Omega)'\Omega^{-1}\tilde{u}_t(\theta;\Omega)\right)$ where

$$
\begin{aligned}
&y_t(\theta) = \triangle y_t + \theta y_{t-1}(\theta),\\
&C_t(\theta) = \triangle C_t + \theta C_{t-1}(\theta),
\end{aligned}
$$

and

$$
\tilde{u}_t(\theta) = y_t(\theta) - C_t(\theta)\left(\sum_{s=1}^{T} C_s(\theta)\Omega^{-1}C_s(\theta)\right)^{-1}
$$

By the Neyman-Perason Lemma, the test rejects for large values of

$$
P(\theta^*;\Omega)_T = \sum_{t=1}^{T}\tilde{u}_t(i;\Omega)'\Omega^{-1}\tilde{u}_t(i;\Omega) - \sum_{t=1}^{T}\tilde{u}_t(\theta^*;\Omega)'\Omega^{-1}\tilde{u}_t(\theta^*;\Omega),
$$

where $i = (1, \cdots, 1)'$. By construction, the test is most powerful test for $\theta = 1$ against the simple specific alternative $\theta = \theta^*$. We are in a position to give the asymptotic distribution of the test statistics under the local alternative:
Theorem: Let y_{it} be generated as described in the preceding section. Suppose $u_t \sim iidN(0,\Omega)$ and $\phi = T(1-\theta) \geq 0,\ \bar{\phi} = T(1-\bar{\theta})$. Then $P(\theta^*;\Omega)_T \rightarrow_d P_\infty(\phi, \bar{\phi}, \rho^2)$, where

$$
\begin{aligned}
P_\infty(\phi, \bar{\phi}, \rho^2) = &\left(\int_0^1 C_{\bar{\phi}}(r)dW_{\bar{\phi}}^{\phi}(r)\right)'\left(\int_0^1 C_{\bar{\phi}}(r)C_{\bar{\phi}}(r)'dr\right)\\
&-\left(\int_0^1 C_0(r)dW_0^{\phi}(r)\right)'\left(\int_0^1 C_0(r)C_0(r)'dr\right)^{-1}\\
&\left(\int_0^1 C_0(r)dW_0^{\phi}(r)\right)\left(\int_0^1 C_{\bar{\phi}}(r)dW_{\bar{\phi}}^{\phi}(r)\right)\\
&-\bar{\phi}^2\lambda^2\int_0^1 U_{\bar{\phi}}^{\phi}(r)^2dr\\
&+2\bar{\phi}^2\lambda^2\left(\int_0^1 U_{\bar{\phi}}^{\phi}(r)dU^{\phi}(r) - \rho\int_0^1 U_{\bar{\phi}}^{\phi}(r)dU(r)\right)\\
&-\left(\int_0^1 C_0(r)dW_0^{\phi}(r)\right)'\left(\int_0^1 C_0(r)C_0(r)'dr\right)'\left(\int_0^1 C_0(r)dW_0^{\phi}(r)\right).
\end{aligned}
$$

in which $\rho \in [0,1)$, and $S^{1/2}$ be the lower Cholesky decomposition of the matrix

$$S(\rho) = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix},$$
$$U_\alpha^\phi(r) = U^\phi(r) - \alpha \int_0^r exp(-\alpha(r-s))U^\phi(s)ds,$$
$$C_\alpha^i(r) = C^i(r) - \alpha \int_0^r exp(-\alpha(r-s))C^i(s)ds,$$

where

$$U^\phi(r) = U(r) + \phi \int_0^r U(s)ds,$$
$$C^i(r) = (1, \cdots, r^p)' \quad \text{for all } i,$$
$$W_\alpha^\phi(r) = S^{-1/2} \begin{pmatrix} U_\alpha^\phi(r) \\ V(r) \end{pmatrix},$$
$$C_\alpha(r) = \begin{pmatrix} C_\alpha^i(r) \\ C^i(r) \end{pmatrix} S^{-1/2\prime},$$
$$\lambda = (1-\rho^2)^{-1/2},$$

and $(U,V)'$ is a Brownian motion with covariance matrix S.

While the expression of the asymptotic distribution of the test statistics appears to be complicated, the basic underlying message emerges from the characterization is simply that the limit distribution depends on the degree of correlation between covariates due to the presence of cross section dependence and the particular individual series under study. We have investigated how the degree of cross section dependence can have an impact on the power performance of the test statistics. Overall the conclusion from our simulations is that the difference in power gain between the tests with and without cross section dependence could be very remarkable as the degree of cross section dependence increases, thus an improvement in the quality of covariates.

4 Self Evaluation

Initially the project was aimed at devising tests for panel unit root with cross section dependence. Along the research journey, to develop tests for panel stationarity appears to be more promising and fruitful, given that there have been so many panel tests for unit root available. Our first part of investigations into the nature of the test statistics of interest generally reveal that the presence of cross section dependence in fact is a merit to have in the data. We show that by incorporating the information in the covariates resulting from the presence of cross section dependence into the construction of the tests, the power gains could be quite substantial for many experiments we consider. However, the research is still far from the ending point. More to be under consideration could be summarized as follows:

1. The asymptotic distribution obtained is derived as T goes to infinity. The type of the approximation will be accurate only when in practice the panel have a long span in time horizon. When T is small and N is large, alternative asymptotic results

might prove to be useful to have as well. It is also important to find out the weak convergence of the test statistics by letting both T and N go to infinity.

2. The test statistics are proposed for any individual series in the panel. However, how to pool the individual test statistic in a hope that the combined test statistics can have a maximum power remains to be an important and significant research agenda. In other words, we are asking if there is an efficient test for panel stationarity null in the presence of cross section dependence. Related to the question is how to eliminate the cross section dependence for the pooled test statistics at least asymptotically. The orthogonalization scheme or the augmentation method suggested to deal with the same problem in testing for panel unit roots are likely to be useful alternatives when considering testing for panel stationarity null.

3. The proposed tests are derived in the simple context of iid errors. But this is not often the case in applications. An extension of the derivation to the asymptotic distributions of the test statistics under more flexible assumption of serially correlated errors should be considered as well. Doing this could be very challenging because it would be difficult to derive the asymptotic distribution of the test in the presence of both cross section dependence and serially correlated errors when both T and N are allowed to go infinity.

References

[1] Chang, Y., 2002, "Nonlinear IV Unit Root Tests in Panels with Cross-Sectional Dependence," *Journal of Econometrics* **11**, 261-292.

[2] Choi, I., 2001, "Unit Root Tests for Panel Data," *Journal of International Money and Banking* **20**, 249-272.

[3] Elliott, G., and M. Jansson, 2002, "Testing for Unit Roots with Stationary Covariates," *Journal of Econometric*, forthcomming.

[4] Elliott, G., T.J. Rothenberg, and J.H. Stock, 1996, "Efficient Tests for an Autoregessive Unit Root," *Econometrica* **64(4)**, 813-836.

[5] Hadri, K., 2000, "Testing for Stationarity in Heterogeneous Panel Data," *Econometrics Journal* **3**, 148-161.

[6] Hansen, B.E., 1992, "Rethinking the Univariate Approach to Unit Root Testing: Using Covariates to Increase Power," *Econometric Theory* **11**, 1148-1171.

[7] Im, K., H. Pesaran, and Y. Shin, 2003, "Testing for Unit Roots in Heterogeneous Panels," *Journal of Econometrics* **115**, 53-74.

[8] Kwiatkowski, D., P.C.B. Phillips, P. Schmidt, and Y. Shin, 1992, "Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root: How Sure are We that Economic Time Series have a Unit Root," *Journal of Econometrics* **54**, 159-178.

[9] Moon, H.R. and B. Perron, 2003, "Testing for a Unit Root in Panels with Dynamic Factors," *Journal of Econometrics*, forthcoming.

[10] Nyblom, J., and A. Harvey, 2000, "Tests of Common Stochastic Trends," *Econometric Theory* **16**, 176-199.

[11] Pesaran, M.H., 2003, "A Simple Panel Unit Root in the Presence of Cross Section Dependence," manuscript, Cambridge University.

[12] Phillips, P.C.B. and D. Sul, 2002, "Dynamic Panel Estimation and Homogeneity Testing Under Cross Section Dependence," *Cowles Foundation Discussion Paper No. 1362*, Yale University.

[13] Saikkonen, P. and R. Luukkonen, 1993a, "Point Optimal Tests for Testing the Order of Differencing in ARIMA Models," *Econometric Theory* **9**, 343-362.

[14] Saikkonen, P. and R. Luukkonen, 1993b, "Testing for a Moving Average Unit Root in Autoregressive Integrated Moving Average Models," *Journal of the American Statistical Association* **88**, 596-601.

[15] Shin, Y., 1994, "A Residual-Based Test of the Null of Cointegration Against the Alternative of No Cointegration," *Econometric Theory* **10**, 91-115.