行政院國家科學委員會專題研究計畫成果報告

In traditional statistical methods, data obtained through questionnaires will be a collection of many single values or some specified interval. However, these data could not totally be represented for person's opinions or feelings. What if respondents could answer questions subjectively by use of membership or the intermediate number of a given interval on the survey? When we are making In fuzzy representation, compared to binary logic, we can feel free to represent our feelings. It must be closer to our real feelings. In the investigation of social science, it would be more realistic and reasonable to use fuzzy modes than one stable unique point, when we are considering problems with ambiguity.

In order to study the patterns of inference when using "statements" which could not be regarded as either true or false, we review in this section the Boolean logic and an example of multi-valued logic, namely the so-called probability logic which is used in artificial intelligence.

If u,v $\in \nu$ then $u', u \wedge v, u \vee v$ are formulas. Note that no meaning has been attached to anything yet. Since variables (or proposition) are either true or false, we considered the truth value space {0,1}. For each map $t : \nu \to \{0,1\}$, we extend is to $\tilde{t} : \ \to \{0,1\}$n a nature way. For example, $\tilde{t}(a \wedge b) = t(a) \wedge t(b)$, once we specify truth tables. For the connective, i.e. define, $\vee, \wedge, '$, on {0,1}. Below are the truth tables of connectives in classical two-valued logic.

$\vee$	**0**	**1**
0	**0**	**1**
1	**1**	**1**

$\wedge$	**0**	**0**
0	**0**	**0**
1	**0**	**1**

	'
0	**1**
1	**0**

Each map $\tilde{t} : \ \to \{0,1\}$ is called a *truth evaluation.* A formula A is called a *tautology* if $\tilde{t}(A) = 1$ for any $t : \nu \to \{0,1\}$.

For example, if $a \in \nu$,then $a \vee a'$ is a tautology.

A formula A logically implies formula B if $\forall \tilde{t}$ such that $\tilde{t}(A) = 1$, then $\tilde{t}(B) = 1$. This important concept can be expressed in terms of a (material) implication operator $A \Rightarrow B$, which is defined as $A' \vee B$, i.e. with truth table

A	B	$A \Rightarrow B$
1	1	1
0	1	1
1	0	0
0	0	1

Thus, A logically implies B if and only if $A \Rightarrow B$ is a tautology. Two formulas A and B are logically equivalent when $\tilde{t}(A) = \tilde{t}(B)$, $\forall t : \nu \to \{0,1\}$. This is the same as saying that $A \Leftrightarrow B$ is a tautology, where by $A \Leftrightarrow B$ we mean $A \Rightarrow B$ and $B \Rightarrow A$.

It is natural to regarded two logically equivalent formulas as equal. The logical equivalence relation is in fact an equivalence relation, and as such it partitions. The space

$/\Leftrightarrow$ is the set of all equivalence classes. If we denoted by [a] the equivalence class containing the formula a, and set $[a]\vee[b]=[a\vee b]$, $[a]\wedge[b]=[a\wedge b]$, $[a]'=[a']$ then $/\Leftrightarrow$ is a Boolean algebra, called the (classical) propositional calculus. The connection between propositional calculus and set theory is left as an exercise (exercise 2).

In summary, propositional calculus is a logical atomic proposition. The validity of arguments does not depend on the meaning of these atomic propositions, but rather on the form of the argument. For example, one inference rule in prepositional calculus, known as modus ponens, states that from $a\Rightarrow b$ and a, we can deduce b logically, i.e. $(a\Rightarrow b)\wedge a\Rightarrow b$ is a tautology.

Traditionally, modes indicate the greatest clustering or concentration of values. It is often used for making manageable, economical, social, educational or political decisions. We will further discuss them later. Fuzzy mode of discrete type is simpler than that of continuous type. Also the computations of discrete type fuzzy mode won't be so complicated than that of continuous type. When questionnaires are vague and elements in factor set are distinguishable, an agreement of specified subject under consideration could be attained by calculating the fuzzy model of discrete type.

Before defining fuzzy modes of discrete, we should first define the fuzzy samples of discrete type.

Important Definitions:

Fuzzy Samples of Discrete Type

Let U denote a universal set, $\{L_j\}_{j=1}^{k}$ be a set of linguistic variables over U, and $\{P_i\}_{i=1}^{n}$ be a fuzzy sample. All combinations of linguistic variables L_j's and fuzzy samples P_i's constitute a fuzzy family of discrete type over U. The fuzzy samples for a subject X is defined as :

$$FFM_X^D=\{(P_i,L_j)\,|\,i=1,\cdots,n\ j=1,\cdots k\}$$

Fuzzy Modes of Discrete Type

Let U denote a universal set and FFM_X^D be the fuzzy family of subject X over U. For each sample P_i, we assign a standardized membership m_{ij} ($\sum_{j=1}^{k}m_{ij}=1$) to every corresponding linguistic variable L_j. Define FI_j to be the fuzzy index to linguistic variable L_j as:

$$\mathrm{FI}_j=\sum_{i=1}^{n}I_{ij}(m_{ij}),\ j=1,\ldots,\mathrm{k} \tag{3.2}$$

, where

$$I_{ij}(m_{ij})=\begin{cases}1, \text{ if } m_{ij}\geq \sim\\ 0, \text{ if } m_{ij}<\sim\end{cases} \tag{3.3}$$

, and ~ is defined to be a significant ~-cut for level set $i(FFM_X^D)$, which is a collection of membership m_{ij}. The fuzzy modes for subject X, FM^D is thus defined to be specified L_j with maximum value of FI_j. That is, $FM_X^D=L_{j*}$,where L_{j*} satisfies $\max_j\{FI_j\}=FI_{j*}$.

From the above definition, we could tell that there might exist more than one fuzzy modes. Under this situation, we could conclude that this subject owns fuzzy modes or it has more than one common opinion.

It will be more complex to discuss fuzzy modes of continuous type. The membership functions of uniform and symmetric shape. When the data is of continuous type, we will first try to separate them into several intervals with equal length, and let respondents choose one among these intervals. For example, if people are asked “What’s your monthly income?” and investigators designed the following five selections: *(1)under NT \$20,000 (2)NT \$20,000~\$40,000 (3)NT \$40,000~\$60,000 (4)NT \$60,000~\$80,000 and (5)over NT \$80,000.* The interval with the largest frequencies is called the modal interval, and the modal interval midpoint is taken as the value of the mode. What if the answers of respondents are multiple choices, or they select the first or the last ones? We may wonder what the truth of respondents is. Therefore, if we use the method of fuzzy modes, we could let respondents choose any possible intervals, and meanwhile they should assign a value to each choice to represent the possibility that it might happen in their real life. We could expect that a reasonable result can be obtained.

Fuzzy Modes of Tri-Type

Let U be a universal set, ~ be a significant ~-cut. If a_j *and* b_j *(*$a_j < b_j$*) are two units of* $«_j^{-1}(\sim)$ *and j=1,2,…,n. Consider the fuzzy samples* FFM_X^{C-Tri} *. Suppose U is partitioned into k distinct area, the degree of fuzziness for each of the fuzzy sets* $A_1, A_2, \ldots, A_k$ *is measured by :*

$$A_i = \sum_{j=1}^{n} \int_{(T_i,T_{i+1})\cap(a_j,b_j)} \left(«_j(X) - \sim\right) d«(X), \quad i = 1,\ldots,k \qquad (3.8)$$

where $T_1, T_2, \ldots, T_k$ *is a partition of R over U and* (T_i, T_{i+1})*'s are intervals for* $A^i s$*'. The fuzzy mode for Tri-type fuzzy samples is defined to be the interval with maximum fuzziness, that is:*

$$FM_X^{C-Tri} = \left\{ (T_i, T_{i+1}) \cap (a_j, b_j) \,\middle|\, \forall j \neq i, A_i > A_j, i, j = 1, 2, \ldots, k \right. \qquad (3.9)$$

The following Figure 1 illustrates the way that the degree of fuzziness is calculated

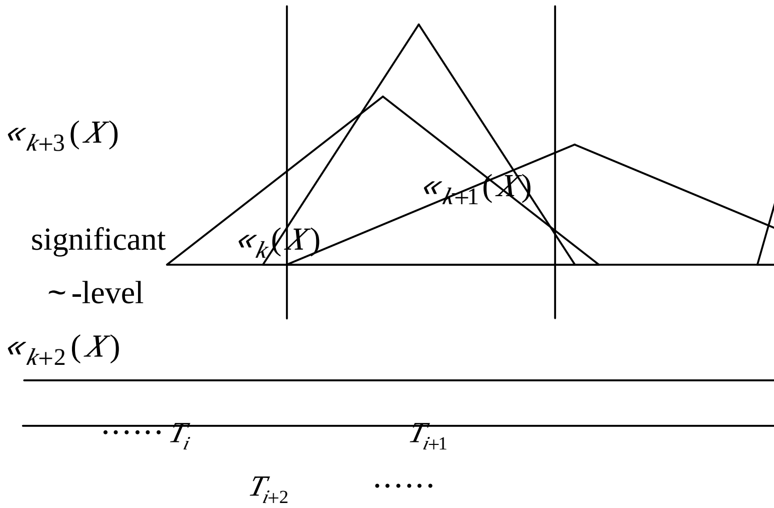

Figure1 Fuzzy memberships of Triangular-shape

Some Properties of Fuzzy Modes

Statistical parameters are used to represent characteristics of a population. However, there still many characteristics which are difficult to be computed by traditional parameters such as expectation, medium, and mode. Especially when we do research on the public opinions in the social science. The concept of fuzzy mode is developed to overcome the limitation of the conventional statistical technologies. Here, we propose some properties of fuzzy mode which can be easily used in the real life. We will also compare these two types of modes with the traditional ones.

Property 1. *Let U be an universal set, $\{L_j\}_{j=1}^{k}$ be a set of linguistic variables over U, and L_l be the traditional mode. For fuzzy mode of discrete type, if the maximum membership m_{ij} for every sample is greater than specified given significant ~-cut and lies on the same linguistic variable L_l, then it must also be the traditional mode.*

Property 2. *Let U be an universal set, $\{P_i\}_{i=1}^{n}$ be a fuzzy sample over U with size n, and m_{ij}s' are standardized memberships of sample P_i of linguistic variable L_j. For the discrete types of fuzzy mode, the fuzzy mode is also the traditional mode for given significant 0.5-cut, if every P_i owns only one m_{ij}>0.5.*

Property 3. *For discrete type of fuzzy mode, if there exists a sample P_i having same maximum memberships for different linguistic variable L_js', we will conclude that there doesn't exist mode. But we could find fuzzy modes for given appropriate significant ~-cut.*

Property 4. *For discrete type of fuzzy mode, we could control the sample size contained in fuzzy mode by changing the signifi*

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