

行政院國家科學委員會專題研究計畫 成果報告

有關反應擴散方程系統的爆破臨界指數

計畫類別：個別型計畫

計畫編號：NSC92-2119-M-004-001-

執行期間：92年10月01日至93年07月31日

執行單位：國立政治大學應用數學學系

計畫主持人：符聖珍

報告類型：精簡報告

處理方式：本計畫可公開查詢

中 華 民 國 93 年 11 月 4 日

中文摘要

本計畫中，我們研究以下系統的爆破臨界指數

$$u_t = \Delta u^m + v^p - au^r, \quad x \in \Omega, t > 0,$$

$$v_t = \Delta v^n + u^q - bv^s, \quad x \in \Omega, t > 0,$$

其中 $m, n > 1$ 。我們找到一個爆破臨界指數，使得當 $pq < \max\{m, r\} \max\{n, s\}$ 時，所有的解均為全域解(global solution); 當 $pq > \max\{m, r\} \max\{n, s\}$ 時，則存在爆破解(blow-up solution)。當 $pq = \max\{m, r\} \max\{n, s\}$ 時，解是否全域則視 m, n, r, s 而定。

Abstract

In this project, we study the blow-up critical exponent for the following system.

$$u_t = \Delta u^m + v^p - au^r, \quad x \in \Omega, t > 0,$$

$$v_t = \Delta v^n + u^q - bv^s, \quad x \in \Omega, t > 0,$$

where $m, n > 1$. We obtain that all solutions exist globally if $pq < \max\{m, r\} \max\{n, s\}$; while there exist blowing up solutions if $pq > \max\{m, r\} \max\{n, s\}$. For the critical case $pq = \max\{m, r\} \max\{n, s\}$, the existence or nonexistence of global solutions depends on the relation between the exponent of m, n, r, s .

Critical exponent for a system of slow diffusion equations with both reaction and absorption terms

Sheng-Chen Fu^{*†}

Department of Mathematical Sciences, National Chengchi University,
64, S-2 Zhi-nan Road, Taipei 116, Taiwan

1 Introduction

In this paper, we consider the following degenerate parabolic system

$$u_t = \Delta u^m + v^p - au^r, \quad x \in \Omega, \quad t > 0, \quad (1.1)$$

$$v_t = \Delta v^n + u^q - bv^s, \quad x \in \Omega, \quad t > 0, \quad (1.2)$$

with boundary condition

$$u = v = 0, \quad x \in \partial\Omega, \quad t > 0, \quad (1.3)$$

and initial condition

$$u(x, 0) = u_0(x), \quad v(x, 0) = v_0(x), \quad x \in \Omega, \quad (1.4)$$

where Ω is a bounded domain in R^N with a smooth boundary $\partial\Omega$, $m, n > 1$, p, q, r, s, a, b are positive constants. It is well-known that when $m, n > 1$ the problem (1.1)-(1.4) admits solutions only in some weak senses. Since we are interested only in nonnegative solutions, we therefore assume that the initial functions u_0 and v_0 are nonnegative and $u_0, v_0 \in L^\infty(\Omega)$.

The system (1.1)-(1.4) can be used to model as the cooperative reaction of two species in an ecological system. The presence of the u -population species encourages the growth of the v -population species but reduces its own growth and vice versa. The choice of $m, n > 1$ describes the density dependent diffusion phenomenon.

We say that the solution (u, v) of the problem (1.1)-(1.4) blows up in finite time if there exists a finite time $T > 0$ such that the solution is defined in $(0, T)$, and

$$\limsup_{t \nearrow T} \{ \|u(\cdot, t)\|_\infty + \|v(\cdot, t)\|_\infty \} = +\infty.$$

^{*}November 3, 2004

[†]*Email address:* denise@math.nccu.edu.tw

The motivation of this study is from [1] in which Bedjaoui and Souplet studied (1.1)-(1.4) for the case $m = n = 1$. By comparing with some suitable supersolutions and subsolutions, they obtained optimal conditions on the exponents p, q, r, s for the existence of blowing up solutions and the global boundedness of all solutions, respectively.

The main purpose of this paper is to study the existence or nonexistence of global solutions for the problem (1.1)-(1.4) when $m, n > 1$. Indeed, we show that all solutions are globally bounded if $pq < \max\{m, r\} \max\{n, s\}$; while there are finite time blowing up solutions if $pq > \max\{m, r\} \max\{n, s\}$ and the initial data are sufficiently large. For the critical case $pq = \max\{m, r\} \max\{n, s\}$, the existence or nonexistence of global solutions depends on the relation between the exponents m, n, r, s , and also the range of the parameters a, b .

We mention here that the problem (1.1)-(1.4) with $a = b = 0$ has been investigated by many authors, see for example and the reference cited therein. Also, the problem for a slow diffusion equation with absorption of the type $u_t = \Delta u^m - au^r$ has been studied extensively (see, for example, [2-8] and the reference cited therein).

This paper is organized as follows. We study the case $pq \neq \max\{m, r\} \max\{n, s\}$ in §2 and the critical case $pq = \max\{m, r\} \max\{n, s\}$ in §3.

2 The case $pq \neq \max\{m, r\} \max\{n, s\}$

For convenience, we denote $\Omega \times (0, T)$ and $\partial\Omega \times (0, T)$ by Q_T and S_T , respectively. Based on [9], we use the following definition of (weak) solution throughout this paper.

Definition 2.1 *A pair of functions (u, v) is called a solution of (1.1)-(1.4) in Q_T , $0 < T < \infty$, if*

(i) $u, v \in L^\infty(Q_T) \cap C([0, T]; L^2(\Omega))$ and $u^m, v^n \in L^2(0, T; H^1(\Omega))$,

(ii) u and v satisfy the identities

$$\begin{aligned} & \int_{\Omega} \varphi(x, T)u(x, T)dx + \iint_{Q_T} \nabla\varphi \cdot \nabla u^m dxdt \\ &= \iint_{Q_T} [\varphi(v^p - au^r) + \varphi_t u] dxdt + \int_{\Omega} \varphi(x, 0)u_0(x)dx, \end{aligned}$$

$$\begin{aligned} & \int_{\Omega} \varphi(x, T)v(x, T)dx + \iint_{Q_T} \nabla\varphi \cdot \nabla v^n dxdt \\ &= \iint_{Q_T} [\varphi(u^q - bv^s) + \varphi_t v] dxdt + \int_{\Omega} \varphi(x, 0)v_0(x)dx, \end{aligned}$$

for any $\varphi \in C^1(\overline{Q_T})$ such that $\varphi = 0$ on S_T .

(iii) $u^m = 0, v^n = 0$ on S_T in the trace sense.

Definition 2.2 *A pair of functions (\bar{u}, \bar{v}) is called a (weak) supersolution of (1.1)-(1.3) in Q_T , $0 < T < \infty$, with initial data (\bar{u}_0, \bar{v}_0) , if*

(i) $\bar{u}, \bar{v} \in L^\infty(Q_T) \cap C([0, T]; L^2(\Omega))$ and $\bar{u}^m, \bar{v}^n \in L^2(0, T; H^1(\Omega))$,

(ii) \bar{u} and \bar{v} satisfy the inequalities

$$\begin{aligned} & \int_{\Omega} \varphi(x, T) \bar{u}(x, T) dx + \iint_{Q_T} \nabla \varphi \cdot \nabla \bar{u}^m dx dt \\ & \geq \iint_{Q_T} [\varphi(\bar{v}^p - a\bar{u}^r) + \varphi_t \bar{u}] dx dt + \int_{\Omega} \varphi(x, 0) \bar{u}_0(x) dx, \end{aligned}$$

$$\begin{aligned} & \int_{\Omega} \varphi(x, T) \bar{v}(x, T) dx + \iint_{Q_T} \nabla \varphi \cdot \nabla \bar{v}^n dx dt \\ & \geq \iint_{Q_T} [\varphi(\bar{u}^q - b\bar{v}^s) + \varphi_t \bar{v}] dx dt + \int_{\Omega} \varphi(x, 0) \bar{v}_0(x) dx, \end{aligned}$$

for any nonnegative function $\varphi \in C^1(\overline{Q_T})$ such that $\varphi = 0$ on S_T .

(iii) $\bar{u}^m \geq 0, \bar{v}^n \geq 0$ on S_T in the trace sense.

A (weak) subsolution is defined by replacing \geq in (ii) and (iii) by \leq .

We shall use the following comparison principle to prove the existence or nonexistence of global solutions. The proof can be found in [9] and we omit it.

Lemma 2.1 *Let (\bar{u}, \bar{v}) and $(\underline{u}, \underline{v})$ be a supersolution and a subsolution of (1.1)-(1.3) in Q_T , $T > 0$, with initial data satisfying $\bar{u}(x, 0) \geq \underline{u}(x, 0)$ and $\bar{v}(x, 0) \geq \underline{v}(x, 0)$. Then $\bar{u} \geq \underline{u}$ and $\bar{v} \geq \underline{v}$ in Q_T .*

We remark here that the global existence of solutions for (1.1)-(1.4) when $p < m$ and $q < n$ has been obtained in [9]. Using the super-sub-solution method, we can also recover this known results. Indeed, we have the following theorem for the subcritical case.

Theorem 2.2 *Suppose that $pq < \max\{m, r\} \max\{n, s\}$. Then all solutions of (1.1)-(1.4) are globally bounded.*

Proof. Take $R > 0$ such that $\Omega \subset B(0; R)$. Let $(\bar{u}, \bar{v}) = (C_1 e^{-L_1|x|^2}, C_2 e^{-L_2|x|^2})$, where L_1, L_2, C_1, C_2 are positive constants satisfying

$$L_1 \leq N/(4mR^2), \quad L_2 \leq N/(4nR^2), \quad C_1 \geq e^{L_1 R^2} |u_0|_\infty, \quad C_2 \geq e^{L_2 R^2} |v_0|_\infty,$$

and

$$\begin{aligned} C_2^{pq} e^{q|rL_1-pL_2|R^2} & \leq a^q C_1^{qr} \leq a^q b^r e^{-r|qL_1-sL_2|R^2} C_2^{rs}, \quad \text{if } pq < rs, \\ C_2^{pq} e^{q|mL_1-pL_2|R^2} & \leq (NmL_1)^q C_1^{mq} \leq (NmL_1)^q (NnL_2)^m e^{-m|qL_1-nL_2|R^2} C_2^{mn}, \quad \text{if } pq < mn, \\ C_2^{pq} e^{q|rL_1-pL_2|R^2} & \leq a^q C_1^{qr} \leq a^q (NnL_2)^r e^{-r|qL_1-nL_2|R^2} C_2^{nr}, \quad \text{if } pq < nr, \\ C_2^{pq} e^{q|mL_1-pL_2|R^2} & \leq (NmL_1)^q C_1^{mq} \leq b^m (NmL_1)^q e^{-m|qL_1-sL_2|R^2} C_2^{ms}, \quad \text{if } pq < ms. \end{aligned}$$

It is easy to check that (\bar{u}, \bar{v}) is a supersolution of (1.1)-(1.3) with $\bar{u}(x, 0) \geq u_0(x)$ and $\bar{v}(x, 0) \geq v_0(x)$. Thus, by Lemma 2.1, we obtain that $u \leq \bar{u}$ and $v \leq \bar{v}$ as long as the solution (u, v) exists. Therefore, (u, v) is globally bounded. This completes the proof. \square

Borrowing an idea from [1] and [10], we will construct a self-similar subsolution to prove the existence of blowing up solutions.

Theorem 2.3 *Suppose that $pq > \max\{m, r\} \max\{n, s\}$. Then the solution of the problem (1.1)-(1.4) blows up in finite time for initial data large enough.*

Proof. We first consider the case $m \leq r$ and $n \leq s$. Without loss of generality, we may assume that $0 \in \Omega$. Since $pq > rs$, we have either $s/q < (p+1)/(q+1)$ or $r/p < (q+1)/(p+1)$. Therefore, we may without loss of generality assume that $s/q < (p+1)/(q+1)$ (otherwise, we exchange the roles of u and v).

We choose λ and β such that

$$\frac{s}{q} < \lambda < \min\left\{\frac{p+1}{q+1}, \frac{p}{r}\right\} \quad \frac{1}{\lambda q - 1} < \beta < \frac{1}{s-1}.$$

Set $\alpha = \lambda\beta$. Then it is easy to check that

$$\beta p > \alpha + 1, \quad \beta p > \alpha r \geq \alpha m, \quad \alpha q > \beta + 1 > \beta s \geq \beta n. \quad (2.1)$$

Pick a positive number l such that

$$l < \frac{1}{2} \min\{\beta p - \alpha m, \beta + 1 - \beta n, \alpha q - \beta n\}. \quad (2.2)$$

We seek a subsolution of the following form

$$\underline{u}(x, t) = (T-t)^{-\alpha} U\left(\frac{|x|}{(T-t)^l}\right), \quad \underline{v}(x, t) = (T-t)^{-\beta} V\left(\frac{|x|}{(T-t)^l}\right),$$

where $U(y) = (A^2 - y^2)_+^{1/m}$, $V(y) = (A^2 - K^{-2}y^2)_+^{1/n}$, $K \in (1, \sqrt{(\beta n + 2l)/(\beta n)})$ is a constant, and $A, T > 0$ are constants to be determined later. To show $(\underline{u}, \underline{v})$ is a subsolution of (1.1)-(1.3), it suffices to show that

$$\begin{aligned} & (T-t)^{-(\alpha+1)}\{\alpha U(y) + lyU'(y)\} + a(T-t)^{-\alpha r}U^r(y) \\ & - (T-t)^{-(\alpha m+2l)}[(U^m)''(y) + (N-1)/y(U^m)'(y)] \\ & \leq (T-t)^{-\beta p}V^p(y) \end{aligned} \quad (2.3)$$

and

$$\begin{aligned} & (T-t)^{-(\beta+1)}\{\beta V(y) + lyV'(y)\} + b(T-t)^{-\beta s}V^s(y) \\ & - (T-t)^{-(\beta n+2l)}[(V^n)''(y) + (N-1)/y(V^n)'(y)] \\ & \leq (T-t)^{-\alpha q}U^q(y) \end{aligned} \quad (2.4)$$

hold pointwisely for $y > 0$, $y \neq A, KA$.

For $y > A$, it is clear that (2.3) holds. For $0 < y < A$, (2.3) is equivalent to

$$\begin{aligned} & (T-t)^{-(\alpha+1)} \left\{ \alpha(A^2 - y^2)^{1/m} - \frac{2l}{m} y^2 (A^2 - y^2)^{1/m-1} \right\} \\ & + 2N(T-t)^{-(\alpha m+2l)} + a(T-t)^{-\alpha r} (A^2 - y^2)^{r/m} \\ & \leq (T-t)^{-\beta p} (A^2 - y^2/K^2)^{p/n}. \end{aligned} \quad (2.5)$$

Since $A^2 - y^2/K^2 \geq A^2(1 - 1/K^2) > 0$, it follows from (2.1) and (2.2) that (2.5) holds provided that T is sufficiently small.

For $y > KA$, it is clear that (2.4) holds. Let $\theta \in (\sqrt{(\beta n)/(\beta n + 2l)}K, 1)$ be fixed. For $\theta A < y < KA$, (2.4) holds provided that

$$\begin{aligned} & \left\{ \beta(A^2 - y^2/K^2) - \frac{2l}{n} (y^2/K^2) \right\} + 2N(T-t)^{\beta+1-\beta n-2l} (A^2 - y^2/K^2)^{1-1/n} \\ & + b(T-t)^{\beta+1-\beta s} (A^2 - y^2/K^2)^{1-1/n+s/n} \leq 0. \end{aligned} \quad (2.6)$$

Since $\beta(A^2 - y^2/K^2) - 2l/n(y^2/K^2) \leq A^2(\beta - \theta^2/K^2(\beta + 2l/n)) < 0$, it follows from (2.1) and (2.2) that (2.6) holds provided that T is sufficiently small. For $0 < y < \theta A$, (2.4) is equivalent to

$$\begin{aligned} & (T-t)^{-(\beta+1)} \left\{ \beta(A^2 - y^2/K^2)^{1/n} - \frac{2l}{n} (y^2/K^2) (A^2 - y^2/K^2)^{1/n-1} \right\} \\ & + 2N(T-t)^{-\beta n-2l} + b(T-t)^{-\beta s} (A^2 - y^2/K^2)^{s/n} \\ & \leq (T-t)^{-\alpha q} (A^2 - y^2)^{q/m}. \end{aligned} \quad (2.7)$$

Since $A^2 - y^2 \geq A^2(1 - \theta^2) > 0$, it follows from (2.1) and (2.2) that (2.7) holds provided that T is sufficiently small.

Now, we fix T so that $(\underline{u}, \underline{v})$ is a subsolution of (1.1)-(1.3). For any $t \in [0, T)$, $\text{supp } \underline{u}(\cdot, t) \subset \text{supp } \underline{v}(\cdot, t) \subset \overline{B(0; KAT^l)} \subset \Omega$ if $A > 0$ is sufficiently small. Hence it follows from Lemma 2.1 that the solution (u, v) of (1.1)-(1.4) blows up in finite time if $u_0 \geq \underline{u}(x, 0)$ and $v_0 \geq \underline{v}(x, 0)$.

For $m > r$ or $n > s$, we shall only consider the case $m > r$ and $n \leq s$ since the proof for the other two cases is similar. Since $\eta^r \leq \eta^m + 1$ if $\eta \geq 0$, the solution of (1.1)-(1.3) is a supersolution of

$$u_t = \Delta u^m + v^p - a(u^m + 1), \quad x \in \Omega, \quad t > 0, \quad (2.8)$$

$$v_t = \Delta v^n + u^q - bv^s, \quad x \in \Omega, \quad t > 0, \quad (2.9)$$

$$u = v = 0, \quad x \in \partial\Omega, \quad t > 0. \quad (2.10)$$

Proceeding all steps in the previous case with a slight modification, we can show that $(\underline{u}, \underline{v})$ is a subsolution of (2.8)-(2.10) and that the solution (u, v) of (1.1)-(1.4) blows up in finite time if $u_0 \geq \underline{u}(x, 0)$ and $v_0 \geq \underline{v}(x, 0)$. Hence the proof is complete. \square

3 The case $pq = \max\{m, r\} \max\{n, s\}$

Following the proofs of Theorems 2.2 and 2.3 with a slight modification, we get the following results for the critical case.

Theorem 3.1 *Let $pq = \max\{m, r\} \max\{n, s\}$.*

- (i) *Suppose that $r > m$ and $s > n$ and suppose also that a and b are sufficiently small. Then the solution of the problem (1.1)-(1.4) blows up in finite time for initial data large enough.*
- (ii) *Suppose that $r \geq m$, $s \geq n$, and suppose also that $a^q b^r \geq 1$. Then all solutions of (1.1)-(1.4) are globally bounded.*

Proof. (i) Without loss of generality, we may assume that $0 \in \Omega$. Since $pq = rs$, we have either $s/q \leq (p+1)/(q+1)$ or $r/p \leq (q+1)/(p+1)$. Therefore, we may without loss of generality assume that $s/q \leq (p+1)/(q+1)$ (otherwise, we exchange the roles of u and v).

Set $\alpha = s/[(s-1)q]$ and $\beta = 1/(s-1)$. It is easy to check that

$$\beta p \geq \alpha + 1, \beta p = \alpha r > \alpha m, \alpha q = \beta + 1 = \beta s > \beta n. \quad (3.1)$$

Pick a positive number l such that

$$l \leq \frac{1}{2} \min\{\beta p - \alpha m, \beta + 1 - \beta n, \alpha q - \beta n\}. \quad (3.2)$$

We seek a subsolution of the following form

$$\underline{u}(x, t) = C_1(T-t)^{-\alpha} U\left(\frac{|x|}{(T-t)^l}\right), \quad \underline{v}(x, t) = C_2(T-t)^{-\beta} V\left(\frac{|x|}{(T-t)^l}\right),$$

where $U(y) = (A^2 - y^2)_+^{1/m}$, $V(y) = (A^2 - K^{-2}y^2)_+^{1/n}$, K, C_1, C_2 are positive constants satisfying

$$1 \leq K \leq \sqrt{\frac{\beta n + 2l}{\beta n}},$$

$$\alpha A^{2/m} C_1 < [A^2(1 - \frac{1}{K^2})]^{p/n} C_2^p < (\beta A^{2/n})^{-p} [A^2(1 - \frac{1}{K^2})]^{p/n} [A^2(1 - \theta^2)]^{pq/m} C_1^{pq} \quad (3.3)$$

and $A, T > 0$ are constants to be determined later. To show $(\underline{u}, \underline{v})$ is a subsolution of (1.1)-(1.3), it suffices to show that

$$\begin{aligned} & C_1(T-t)^{-(\alpha+1)} \{\alpha U(y) + lyU'(y)\} - C_1^m(T-t)^{-(\alpha m+2l)} \Delta(U^m)(y) \\ & + aC_1^r(T-t)^{-\alpha r} U^r(y) \\ & \leq C_2^p(T-t)^{-\beta p} V^p(y) \end{aligned} \quad (3.4)$$

and

$$\begin{aligned}
& C_2(T-t)^{-(\beta+1)}\{\beta V(y) + lyV'(y)\} - C_2^m(T-t)^{-(\beta n+2l)} \Delta(V^n)(y) \\
& \quad + bC_2^s(T-t)^{-\beta s}V^s(y) \\
& \leq C_1^q(T-t)^{-\alpha q}U^q(y)
\end{aligned} \tag{3.5}$$

hold pointwisely for $y > 0$, $y \neq A, KA$, where $\Delta = d^2/dy^2 + (N-1)/yd/dy$.

For $y > A$, it is clear that (3.4) holds. For $0 < y < A$, (3.4) is equivalent to

$$\begin{aligned}
& C_1(T-t)^{-(\alpha+1)} \left\{ \alpha(A^2 - y^2)^{1/m} - \frac{2l}{m}y^2(A^2 - y^2)^{1/m-1} \right\} \\
& \quad + 2NC_1^m(T-t)^{-(\alpha m+2l)} + aC_1^r(T-t)^{-\alpha r}(A^2 - y^2)^{r/m} \\
& \leq C_2^p(T-t)^{-\beta p}(A^2 - y^2/K^2)^{p/n}.
\end{aligned} \tag{3.6}$$

Since $A^2 - y^2/K^2 \geq A^2(1 - 1/K^2) > 0$, it follows from (3.1), (3.2), and (3.3) that (3.6) holds provided that a and T are sufficiently small.

For $y > KA$, it is clear that (3.5) holds. Let $\theta \in (\sqrt{(\beta n)/(\beta n + 2l)K}, 1)$ be fixed. For $\theta A < y < KA$, (3.5) holds provided that

$$\begin{aligned}
& C_2 \left\{ \beta(A^2 - y^2/K^2) - \frac{2l}{n}(y^2/K^2) \right\} + 2NC_2^n(T-t)^{\beta+1-\beta n-2l}(A^2 - y^2/K^2)^{1-1/n} \\
& \quad + bC_2^s(T-t)^{\beta+1-\beta s}(A^2 - y^2/K^2)^{1-1/n+s/n} \leq 0.
\end{aligned} \tag{3.7}$$

Since $\beta(A^2 - y^2/K^2) - 2l/n(y^2/K^2) \leq A^2(\beta - \theta^2/K^2(\beta + 2l/n)) < 0$, it follows from (3.1), (3.2), and (3.3) that (3.7) holds provided that b and T are sufficiently small.

For $0 < y < \theta A$, (3.5) is equivalent to

$$\begin{aligned}
& C_2(T-t)^{-(\beta+1)} \left\{ \beta(A^2 - y^2/K^2)^{1/n} - \frac{2l}{n}(y^2/K^2)(A^2 - y^2/K^2)^{1/n-1} \right\} \\
& \quad + 2NC_2^m(T-t)^{-\beta n-2l} + bC_2^s(T-t)^{-\beta s}(A^2 - y^2/K^2)^{s/n} \\
& \leq C_1^q(T-t)^{-\alpha q}(A^2 - y^2)^{q/m}.
\end{aligned} \tag{3.8}$$

Since $A^2 - y^2 \geq A^2(1 - \theta^2) > 0$, it follows from (3.1), (3.2), and (3.3) that (3.8) holds provided that b and T is sufficiently small.

Now, we fix T so that $(\underline{u}, \underline{v})$ is a subsolution of (1.1)-(1.3). For any $t \in [0, T)$, $\text{supp } \underline{u}(\cdot, t) \subset \text{supp } \underline{v}(\cdot, t) \subset \overline{B(0; KAT^l)} \subset \Omega$ if $A > 0$ is sufficiently small. Hence it follows from Lemma 2.1 that the solution (u, v) of (1.1)-(1.4) blows up in finite time if $u_0 \geq \underline{u}(x, 0)$ and $v_0 \geq \underline{v}(x, 0)$.

(ii) Let $(\bar{u}, \bar{v}) = (C_1, C_2)$, where C_1 and C_2 are positive constants satisfying $C_1 > |u_0|_\infty$, $C_2 > |v_0|_\infty$, and $C_2^{pq} \leq a^q C_1^{qr} \leq a^q b^r C_2^{rs}$. It is easy to check that (\bar{u}, \bar{v}) is a supersolution of (1.1)-(1.3) with $\bar{u}(x, 0) \geq u_0(x)$ and $\bar{v}(x, 0) \geq v_0(x)$. Thus, by Lemma 2.1, we obtain that $u \leq \bar{u}$ and $v \leq \bar{v}$ as long as the solution (u, v) exists. Therefore, (u, v) is globally bounded. This completes the proof. \square

References

- [1] N. Bedjaoui and P. Souplet, Critical blowup exponents for a system of reaction-diffusion equations with absorption, *Z. Angew. Math. Phys.*, to appear.
- [2] N. D. Alikakos and R. Rostamian, Stabilization of solutions of the equation $\partial u/\partial t = \Delta\varphi(u) - \beta(u)$, *Nonlinear Anal.* **6** (1982), 637-647.
- [3] M. Bertsch, T. Nanbu and L. A. Peletier, Decay of solutions of a degenerate nonlinear diffusion equation, *Nonlinear Anal.* **6** (1982), 539-554.
- [4] M. A. Herrero and J. L. Vazquez, The one-dimensional nonlinear heat equation with absorption: regularity of solutions and interfaces, *SIAM J. Math. Anal.* **18** (1987), 149-167.
- [5] A. S. Kalashnikov, The propagation of disturbances of nonlinear heat conduction with absorption, *USSR Comp. Math. Math. Phys.* **14** (1974), 70-85.
- [6] S. Kamin and L. A. Peletier, Large time behaviour of solutions of the porous media equation with absorption, *Israel. J. Math.* **55** (1986), 129-146.
- [7] , The behaviour of the solutions of degenerate quasi-linear parabolic equations as $t \rightarrow \infty$, *Acta Math. Acad. Sci. Hungar.* **34** (1979), 157-163.
- [8] B. F. Knerr, The behaviour of the solutions of the equation of nonlinear heat conduction with absorption in one dimension, *Trans. Amer. Math. Soc.* **249** (1979), 409-424.
- [9] L. Maddalena, Existence of global solution for reaction-diffusion systems with density dependent diffusion, *Nonlinear Anal.* **8** (1984), 1383-1394.
- [10] P. Souplet and F. B. Weissler, Self-similar subsolutions and blowup for nonlinear parabolic equations, *J. Math. Anal. Appl.* **212** (1997), 60-74.

計畫成果自評

如同先前所預期的，我們可找到一個爆破臨界指數，使得當指數 pq 小於臨界指數時，所有的解均為全域解(global solution); 當指數 pq 大於臨界指數時，則存在爆破解(blow-up solution)。而當指數 pq 等於臨界指數時，目前得到在某些情況下，解是全域的，在某些情況下，解會產生爆破現象。這部分的問題尚未完全解決，本人在日後將繼續鑽研。