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彩券市場之設計、微結構及宏觀行為：代理人基計算經濟建模之應用

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行政院國家科學委員會專題研究計畫成果報告

彩券市場之設計、微結構及宏觀行爲: 代理人基計算經濟建模之應用

Lottery Markets-Design, Micro-Structure, and Macro-Behavior: An ACE Approach

計畫編號: **NSC 93-2415-H-004-005**

執行期限: **93**年**8**月**1**日至**94**年**7**月**31**日

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Abstract

This paper proposes an agent-based computational model of a lottery market based on an expected-utility paradigm, in which agents' decisions regarding lottery participation are based on their own subjective beliefs, and those beliefs are evolving over time with genetic algorithms. The simulation results are then compared with another agent-based lottery market with different agent engineering. It is found that almost all emergent properties, such as the Laffer curve, the halo effect (lottomania), conscious-selection behavior, and the interdependent preference (regretting effect) are qualitatively robust with these two different designs of agents.

Keywords: Agent-Based Computational Model, Lottery Market, Expected-Utility Paradigm, Emergent Property, Designs of Agents

Motivation and Literature Review

(1) pioneered an agent-based computational model of the lottery market. Using the model, one can simulate and analyze different consequences of different designs of lottery markets. For example, the specific question addressed in that paper concerned the *optimal lottery tax rate*, and, as one of their main experiments demonstrated, the optimal tax rate was found to be around 40%, which interestingly mimicked the empirical observations. In this paper, we propose a different behavioral model of gamblers (lottery participants), and compare the agent-based simulation results derived with those obtained in (1). The motivation for this study is two-fold.

Firstly, there has recently been an effort made to understand and further the transferability of knowledge between models and beyond, which is also known as *model-to-model* analysis.[1] Understanding complex adaptive systems often seems to necessitate the use of more than one model. Taking the lottery market as an example, there is generally no unique description of gamblers' behavior as to their decision formation regarding lottery purchasing. The decision can be spontaneous, but it can also be deliberate. Therefore, it would be useful to compare different behavioral models of lottery participants so as to facilitate a better view of what modeling brings to the understanding of real or artificial societies. In this paper, we shall propose a behavioral model of lottery participants based on the *expected utility paradigm*. The agent-based lottery market built upon this agent engineering is then compared with the earlier one built upon a different engineering, namely, *evolutionary fuzzy decision rules*.

The comparison is not just confined to the simulation results, or their fitness to empirical observations. Even though the simulation results of two different agent-based models are the same, different models can still provide different interpretations and hence bring different insights in regard to the same phenomena. For example, as we shall see in this paper, through the expected utility paradigm, the subjective belief of lottery participants is introduced into the model, and one can then explore the role of the evolution of subjective beliefs in the emergent dynamics of lottery dynamics, a mechanism which may not be available in the other agent-based model.

Secondly, despite their interesting finding, (1) do encounter one major weakness, i.e. the absence of the *halo effect*.[2] One possible explanation for the absence of the

[1] See, for example, the entire issue of *Journal of Artificial Societies and Social Simulation* (**JASSS**), Vol 6, No. 4, 2003.

[2] The phenomenon that sales following a rollover are higher than sales prior to the rollover is known in the industry

halo effect is that the agents' behavior in the original model, which is mainly characterized by *evolutionary fuzzy decision rules*, may be a little overwhelming in the sense that it leaves agents with too many parameters to learn. With this degree of sophistication, it becomes difficult for agents to make sense of the simple relationship between the *jackpot size* and *lottery participation*, which may indeed not have any simple rationale behind it. Therefore, in this paper, we try to model agents in a simpler way, which is very much motivated by the standard *expected-utility paradigm*. We are well aware that the conventional expected-utility maximization will not entice agents to purchase lottery tickets, because the expected return is normally negative for lottery investment ((4)). Nevertheless, if agents have expectations *different* from the objective odds of winning, then it is still possible to observe lottery participation. Furthermore, given agents' subjective beliefs, when the jackpot size increases, agents will generally intensify their lottery participation.

Nonetheless, this new device does not necessarily imply that the halo effect or lottomania will occur, as one may expect. As mentioned earlier, this new approach depends heavily upon agents' subjective beliefs, which shall evolve over time. Consequently, agents' subjective beliefs may change during rollover so as to decrease their lottery participation, which makes the net effect difficult to predict.[3]. So, it still makes us wonder whether the phenomenon of lottomania will emerge in this situation.[4]

The remainder of the paper is organized as follows. Section 2 gives the skeleton of the agent-based lottery market used in this paper, whereas the companion algorithm (genetic algorithm) used to implement the lottery market is detailed in Section 3. The experimental designs are given in Section 4 followed by an analysis of the simulation results and the work of comparison. Concluding remarks are given in Section 6.

as the *halo effect* ((3; 4; 5)). The halo effect is partially due to considerate media attention paid to rollovers, which in turn creates a bout of *lottomania*.

[3] A common feature of lotteries is that, if there are no winners in a given draw, the jackpot prize pool from that draw is added to the pool for the next draw, referred as to a *rollover*

[4] As has been well argued in (1), lottomania is not an exogenous setting, but is endogenously generated because one has no control over the size of the jackpot, which in turn is an aggregate outcome of individuals' decisions regarding lottery participation. They further provide an explanation as to why this may occur given the genetic-algorithm learning. See their Section 6.

Agent-Based Modeling of the Lottery Market

We shall follow (1) to build up the three main aspects of lottery behavior, which are *lottomania*, *conscious selection*, and *aversion to regret*. The difference between this version of the agent-based lottery market and the early one in (1) mainly lies in the formation of the decision regarding *lottery participation*, denoted by α, which is defined as the percentage of the income spent on lottery purchasing. In (1), it is formulated as a standard fuzzy decision rule or, more precisely, the *Sugeno style of fuzzy inference*. The *antecedent* is a linguistic description of the jackpot size and is fuzzy, whereas the *consequent* is a numerical description of the lottery investment and is crisp. While this modeling explicitly relates the participation decision to the jackpot size, the preference (utility) and subjective belief of winning is not involved. The current version differs from the former one by taking into account these fundamentals of agents, and modeling the lottery decision within an *expected-utility*, but not necessarily a maximization, framework. The lottery participation is considered to be a result of satisfying behavior. This different agent engineering is introduced as follows.

We begin by working on agents' subjective beliefs regarding the probability of their winning the jackpot. Let p_i be agent i's *subjective belief* (probability) that he will win the jackpot. We index p_i by t as $p_{i,t}$ when we wish to emphasize its adaptation and dynamics. With the device of the expected utility, agents' decisions regarding lottery participation α_i can be treated as a control variable to solve the following equation:

$$u_i(I) = (1-p_i)u_i[(1-\alpha_i)I]+p_iu_i[(1-\alpha_i)I+J], \quad (1)$$

where J denotes the size of the jackpot and I denotes the income the agents received in each period.[5] (1) assume all agents are risk-neutral, and $u(I) = I$. In this paper, we follow a more standard formulation in finance, i.e. by assuming that agents are *risk-averse* and

[5] The idea of Equation (1) is based upon the assumption that the Lottery itself can be *fun*, and hence can enhance utility. ((4)) Based on this assumption, the expected utility of agents who participate in the lottery game is something like $u_i(I) + \epsilon$ and is higher than $u_i(I)$, which is the utility of the non-participating agents. We, however, do not pursue a further maximization problem here due to the difficulty of modeling the detailed relationship between lottery participation and subjective beliefs. We also notice that this is not a precise formulation of the expected utility since there are several different prizes associated with different winning probabilities. For simplicity, only the jackpot prize is considered in this paper. In addition, income I is exogenously given, and is identical for all agents. In a separate paper, (2) study the case where income is heterogeneous among agents, and examine the effect of income distribution on lottery participation.

hence $u(I) = \log I$. It is then easy to show that the solution to problem (1) is

$$\alpha_i^* = \frac{I - (\frac{I}{J^{p_i}})^{\frac{1}{1-p_i}}}{I} = 1 - \frac{(\frac{I}{J^{p_i}})^{\frac{1}{1-p_i}}}{I}. \qquad (2)$$

Clearly, α_i^* is less than 1, but we have to set 0 as a lower bound (corner solution) in case the solution from (2) is less than 0.

Following the discussion of (1), the final *fitness function* after taking into account a psychological factor, namely, *aversion to regret* (θ_i), is then

$$u_i(I) = \begin{cases} u_i[(1-\theta_i)I], & if \ \ \alpha_i^* = 0 \ \ and \ \ N_x > 0, \\ u_i[(1+\theta_i)I], & if \ \ \alpha_i^* = 0 \ \ and \ \ N_x = 0, \\ u_i[(1-\alpha_i^*)I + \pi_i], & otherwise. \end{cases} \qquad (3)$$

The fraction θ_i ($0 \leq \theta_i \leq 1$) in the utility function (3) measures how regretful the non-participant, characterized by $\alpha_i^* = 0$, would be if the jackpot was drawn i.e. the number of winners of the jackpot is positive ($N_x > 0$). On the other hand, in a way that is opposite to regret, the non-gamblers may also derive pleasure from the gamblers' misfortune, in particular when the jackpot is not drawn ($N_x = 0$). Since the mass media generally only give a large coverage to the jackpot winners, and not the losers, we therefore assume that the regret effect is asymmetric between lottery non-participants and participants, as shown in the last part of Equation (3), where π_i is the lottery prize.

The last feature of our model of agents is the *conscious selection*, which refers to non-random selections of combinations of numbers. In an "x/X" lottery game, both a gambler and the lottery agency shall pick x numbers out of a total of X numbers. To take conscious selection into account, let $\vec{b}_i$ be an X-dimensional vector, whose entities take either "0" or "1". Consider a number z, where $1 \leq z \leq X$. If "0" appears in the respective zth dimension, that means the number z will not be consciously selected by the agent, while "1" indicates the opposite. If $\vec{b}_i$ has exactly x 1s, then one and only one combination is defined, and the agent would select only that combination while purchasing the lottery ticket(s). If $\vec{b}_i$ has more than x 1s, then many more combinations can be defined. The agent will then randomly select from these combinations, while purchasing the ticket(s). Finally, if $\vec{b}_i$ has less than x 1s, then those designated numbers will appear on each ticket bought by the agent, whereas the rest will be randomly selected from the non-designated numbers.

All these three aspects of lottery behavior will adapt and change over time; therefore, they are all indexed by t as $p_{i,t}, \theta_{i,t}$, and $\vec{b}_{i,t}$. The evolution of the lottery behavior is then driven by genetic algorithms.

Genetic Algorithms

The standard genetic algorithm is applied to evolving the three behavioral parameters of the entire population, $POP_t \equiv \{p_{i,t}, \vec{b}_{i,t}, \theta_{i,t}\}_{i=1}^N$, where N is the number of agents. GA starts by encoding the behavior parameters to binary digits (binary coding) or real numbers (real coding), usually called the *chromosome* (*finite-string*) representation. Here, we apply *real coding* to $p_{i,t}$ and $\theta_{i,t}$, since they are real numbers between 0 and 1. However, binary coding is applied to $\vec{b}_{i,t}$ given that it is a binary vector.

Tournament selection, associated with a *tournament size* φ, is employed as the selection scheme to determine who are the celebrities (mating pool). Given the mating pool, offspring are generated by applying the two genetic operators: crossover and mutation. First, the *crossover*. Since each chromosome represents the three different aspects of agents' behavior, the crossover is made in a pair-by-pair manner, i.e. to restrict the swap only to the paired characteristic, called a *paired crossover*. Each time the crossover works on only one of the three pairs which are determined randomly. If it is the pair of $\vec{b}_{i,t}$, the one-point crossover with a crossover rate P_c is applied. If it is $p_{i,t}$ or $\theta_{i,t}$, the arithmetic crossover with the same crossover rate is applied. Second, the *mutation*. After the crossover, each part of the resultant chromosome has a chance of being mutated. For $\vec{b}_{i,t}$, the bit mutation with a mutation rate P_m is applied, whereas for $p_{i,t}$ or $\theta_{i,t}$, an arithmetic mutation adjusted to a bit-mutation equivalent is applied to these real parameters as in Equation (4).[6]

$$p^{new} = p^{old} + \sum_{i=1}^{16} B_{P_m} (\frac{1}{2})^i \cdot (-1)^{B_{\frac{1}{2}}}, \qquad (4)$$

where p_i^{old} and p_i^{new} indicate the subjective belief *before* and *after* mutation. B_{P_m} and $B_{\frac{1}{2}}$ are the Bernoulli random variables with success probability P_m (the mutation rate) and one half, respectively.

When all offspring are produced, the steady-state replacement with the generation gap η is applied to replace the old generation. With the parameter η, the agents belonging to the top $1-\eta$ percent would remains and only the agents belonging to the bottom η percent would be replaced by offspring.

Experimental Designs

We would like to see how this new design may come up with anything significantly different from what was

[6] As the arithmetic crossover, we could use the standard arithmetic mutation for the real-coding chromosomes. The reason for using this bit-mutation design is to make our results comparable to those of (1).

Table 1: The Experimental Design

Market Parameters	
Pick x from X (x/X)	5/16
Lottery Tax Rate (τ)	10%, ..., 90%
$s_0, s_1,$ s_2, s_3, s_4, s_5	0%, 0%, 35%, 15%, 12%, 38%
Drawing Periods ($\bar{r}$)	3
Number of Agents (N)	5000
Income (I)	200
GA Parameters	
Range of $p_{i,0}$	[0, 0.003]
Periods (Generations) (T)	500
Crossover Rate (P_c)	90%
Mutation Rate (P_m)	0.1%
Arithmetic Mutation Size	Equation (4)
Tournament Size (φ)	200
Generation Gap (η)	100

obtained in (1). The behavior of the lottery market studied in (1) includes the optimal lottery tax rate associated with a simulated Laffer curve[7], the impact of the regret effect upon the Laffer curve, and the statistical relationship between rollovers. Rollovers usually enhance the attractiveness of the next draw, called the *rollover draw*, *sales* (the halo effect), the evolution of conscious-selection behavior, and the inter-dependence preference (the aversion to regret).

To be able to compare the results of our new design with those of (1), we follow almost the same experimental design as theirs (see Table 1).[8] The design is composed of two parts, namely, *market parameters* and *GA parameters*.

Let us start with the market parameter. In an "x/X" lottery game, both a gambler and the lottery agency shall pick x numbers out of a total of X numbers, and then different prizes are set for different numbers matched. Let y denote the numbers matched. Clearly, $y = 0, 1, ..., x$. Let S_y be the *prize pool* reserved for the winners who matched y numbers. A special term is given to the largest pool, S_x, namely the *Jackpot*.

Each prize pool, S_y, shall be shared by all players who match y numbers, say N_y. The prize pool is defined by the *lottery tax rate*, τ, which is the proportion of sales that is not returned as prizes. Thus, the overall prize pool is $(1-\tau)S$, where S is sales revenue and $1-\tau$ is also called the *pay-out rate*. The overall prize pool will then be distributed to each separate pool based on a distribution $(s_0, ..., s_x : \sum_0^x s_y = 1)$, i.e. $S_y = s_y(1-\tau)S$. In the event where $N_y = 0$, S_y is added to the next draw. A particularly interesting case is $N_x = 0$, i.e. the premise for a rollover draw. It is anticipated that s_y will be increasing in y. To recap, a lottery game can be represented by the following x+4-tuple vector:

$$\mathcal{L} = (x, X, \tau, s_0, ..., s_x),$$

which is also shown in the upper half of Table 1.

Turning to the parameters pertaining to GA, most parameters are the same for those in (1) except for the initial subjective belief, which is not applicable in their model.[9] Here, the initial subjective belief $p_{i,0}$ is uniformly sampled from the range $[0, 0.003]$. This value is set in accordance with the objective probability of winning the jackpot, which is $1/\binom{16}{5} \approx 0.0002$. We then build a range centering around this objective probability. The range starts from 0 and, after a few trials, we find 0.003 to be a reasonable upper limit.[10]

Simulation Results

Twenty five independent runs were conducted based on the design as indicated in Table 1. The results presented below are therefore not based on a single run, but an statistics for these 25 runs. To show the distribution of the 25 runs, in some cases, the box-and-whisker plot is used to demonstrate the statistics, say, the means, of the 25 runs (each run has 500 observations). The point appearing inside each box typically represents the median of this sample distribution, and to make it more visible, these points are connected by a line.

[7] The Laffer Curve is named after the economist Arthur Laffer. He was an advisor to President Reagan in the early 1980s. The Laffer curve suggests that, as taxes increased from fairly low levels, tax revenue received by the government would also increase. However, as tax rates rose, there would come a point where people would not regard it worth working so hard. This lack of incentives would lead to a fall in income and therefore a fall in tax revenue.

[8] However, since the modeling of lottery participation is different between the two, not all control parameters are applicable here. For example, the control parameters pertaining to their fuzzy decision rules are not applicable to our case. Similarly, our parameters as to the belief formation and adaptation is also not applicable to their case.

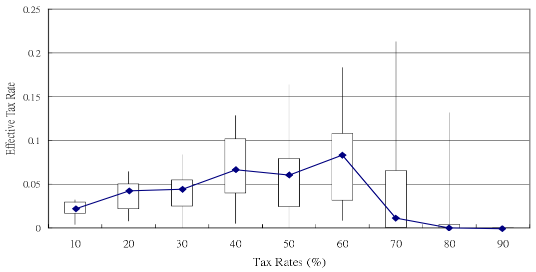

Figure 1: Tax Revenue Curve (Laffer Curve) and the Associated Box-Whisker Plot

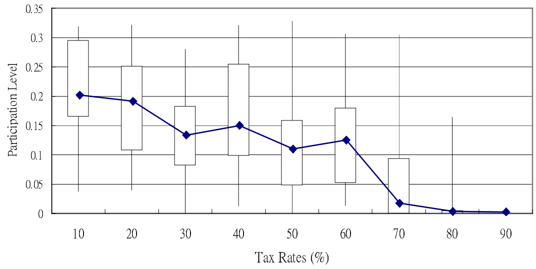

Figure 2: Lottery Participation Rate and the Tax Rate

Lottery Tax Rate and Tax Revenue

Figure 1 depicts the *Laffer curve* phenomenon noticed by (1).[11] The result here is comparable to the one observed earlier.[12] A noticeable difference is the location of the peak. For (1), it is located at a tax rate of 40%, but this one is higher, reaching up to 60%. However, by running a non-parametric test for these medians, it is found that the effective tax rates corresponding from $\tau = 0.4$ to $\tau = 0.6$ are statistically insignificant. In addition to the "shift" on the peak, we also see the change in the distribution of the effective tax rate. By looking at the range (the box-and-whisker plot) of the effective tax rate, there is generally an observed downward tendency.[13]

The appearance of the Laffer curve basically portrays the two counterbalancing forces as shown in Equations (5) and (6).

$$T = \tau S = \tau(\alpha I) \tag{5}$$

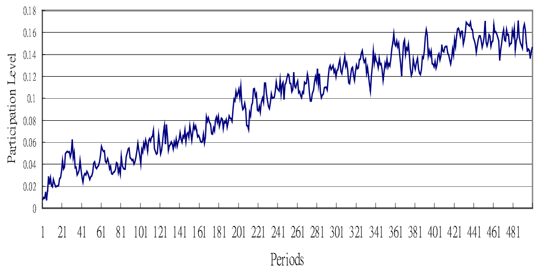

Figure 3: Time Series of the Lottery Participation Rate

$$\frac{\partial T}{\partial \tau} = \underbrace{S}_{+} + \tau \underbrace{\frac{\partial \alpha}{\partial \tau}}_{-} I \tag{6}$$

The positive force as characterized by the plus sign in Equation (6) says that, given the sales, the higher the lottery tax rate, the higher will be the tax revenue. On the other hand, we *expect* that a negative sign for the relationship between lottery participation α and lottery tax rate τ, i.e.

$$\frac{\partial \alpha}{\partial \tau} < 0.$$

Figure 2 confirms this expectation as it does show that the lottery participation rate (α) declines with the lottery tax rate.

Belief and Participation

The distinguishing feature of this paper is that it incorporates agents' subjective beliefs into their decision formation, while the subjective belief is not updated in a Bayesian manner but is based on a social learning style driven by GA. It is, therefore, interesting to see the connection between *agents' beliefs* and *participation*. Figure 3 is a time-series plot of the average lottery participation over all 225 (9×25) runs by pooling together the cases of 9 different tax rates, whereas Figure 4 is the corresponding time-series plot of beliefs. These two figures together present the evolution of lottery participation and beliefs. The two figures support each other

[9] This by no means says that the value of these parameters is unimportant. However, it is not the current focus of this paper.

[10] Obviously, this range cannot be set too high or too low. If it is too low, the subjective belief is not much different from the objective probability, and will result in almost no participation. On the other hand, if it is set too high, the participation will be so high that rollover with a noticeable jackpot size is infeasible. Besides, it may seem unlikely that we can have so many over-confident people in the real world. The other possibility which we have thought about, but have not given it a try, is to replace the uniform distribution with a right-skewed distribution over [0,1], such as a *Beta* distribution with appropriate parameters.

[11] The statistics shown in Figure 1 are derived by dropping the first 100 observations.

[12] Ibid, Figure 8.

[13] So far, we do not have a good explanation for what may cause this observed difference, except to confirm once again the lesson that *models of adaptation matter*. Apart from that, we should point out that agents in our model are risk-averse, whereas agents in (1) are risk neutral. To what extent risk attitude can impact lottery tax revenue is a separate issue which deserves another independent study.

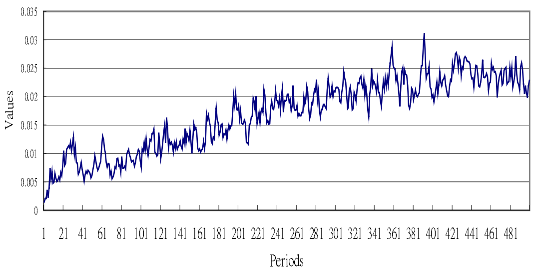

Figure 4: Time Series of the Market Belief (Average of Subjective Beliefs)

with a simultaneous upward tendency. Remember that we start with a very narrow range of the initial subjective belief (see Table 1), but then we watch the emergence of *social euphoria*: the subjective belief gradually evolves toward a more optimistic state. This social euphoria is supported by and then provides feedback to the lottery participation. As shown in Figure 3, α increases from a level of less than 5% to a range of between 12% and 15%, when the subjective belief p rises from the original niche of 0.003 to a level of 0.023.

The result of *social euphoria* deserves a discussion. It is true that most people, most of the time, do not win the jackpot, and there is no prior reason why they should collectively learn to be more optimistic. However, the essence of social learning is that people not only learn from their own experiences, but also from others' experiences. Furthermore, the survival of the fittest principle operating in GA tends to make agents learn more from those who win the lottery, particularly those who win the jackpot (super rich people) than from those who do not (mediocre people). Therefore, with this reinforcing process, individuals' experiences may become minor in updating agents' subjective beliefs.[14]

Rollovers and Sales

It is generally assumed that the large sizes of the rollovers will enhance the attractiveness of the lottery game. Statistics also tell us that the mean sales conditional on the rollover draw is normally *higher* than that of the regular draw.[15] Nevertheless, this phenomenon is not able to be fully replicated in (1).[16] This becomes a

Table 2: Rollover and Sales: Statistics from the Simulated Data

Tax Rates	t statistic (p-value)	α_1 (p-value)	R^2	Anomalies
0.1	-97.277 (0.0000)	0.2507 (0.0000)	0.0546	61.8%
0.2	-94.6081 (0.0000)	0.4593 (0.0000)	0.1289	62.8%
0.3	-86.6937 (0.0000)	0.3447 (0.0000)	0.0898	56.9%
0.4	-97.808 (0.0000)	0.4498 (0.0000)	0.1029	60.2%
0.5	-74.3719 (0.0000)	0.4124 (0.0000)	0.0782	45.5%
0.6	-80.9661 (0.0000)	0.3617 (0.0000)	0.0701	57.7%
0.7	-56.3219 (0.0000)	0.4885 (0.0000)	0.0746	23.3%
0.8	-21.4805 (0.0000)	0.2484 (0.0000)	0.0759	15%
0.9	-3.5939 (0.0049)	0.6067 (0.0000)	0.555	0.4%

The statistics in this table are based on the last 400 observations.

puzzle, and they refer to it as the *disappearance of the halo effect*. A conjecture of this failure has been given in the introductory section of the paper, which also motivates a different design of the agent's behavior in this paper. However, would this new design be able to deliver the halo effect?

To answer this question, we perform similar statistical tests as to what (1) have done, and these are shown in Table 2. The t statistic shown in the second column is a test statistic for the null that the mean sales of the rollover draw is greater than that of the regular draw, i.e. the halo effect exists. From the corresponding p value, we can see that the halo effect is uniformly rejected. This result is consistent with what was found in (1): *the halo effect is again absent*.[17]

However, sales may actually fall in some rollover draws, and the frequency of this *anomaly* can be as high as 20% to 25% in some countries.[18] However, the frequency of anomalies found in (1) is around 60% (when τ=0.4, 0.5), which is simply too high to be comparable with the real data. Here, we encounter a similar problem. The fifth column of Table 2 also indicates the high frequency of anomalies, say 60.2% (τ=0.4), 45.5% (τ=0.5), and 57.7% (τ=0.6). However, a few exceptions

[14] The emergence of social euphoria may also help explain why the lottery participation rate is not so sensitive to medium and high lottery tax rates, such as those ranging from $\tau = 0.4$ to 0.6 shown in Figure 2.

[15] For a review of the empirical evidence, please see Table 3 of (1).

[16] They have carried out two statistical tests for their simulated data. The first is simply to test the difference in mean sales of a rollover draw and a regular draw, and the null that the mean sales of the rollover draw is greater than that of the regular draw is *rejected* for all tax rates. However, conditional upon the rollover draw, it is found that sales do go up with the jackpot size.

[17] At this moment, we have not figured out a compelling reason for the failure to generate the halo effect again except for "murmuring" something as (1) have done. See their Section 6.

[18] See Table 3 of (1).

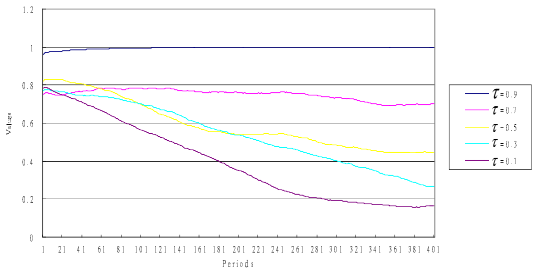

Figure 5: Time Series of Rollover Frequency under Different Tax Rates

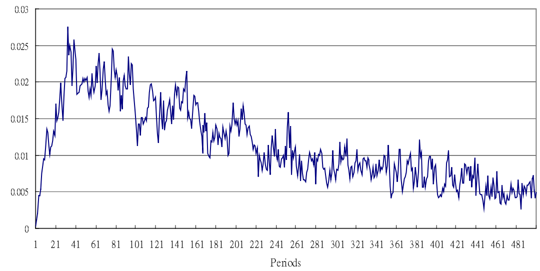

Figure 6: Time Series of Rollover Size, Normalized by Total Income

arise from the case where τ is high. For example, when τ=0.7, the frequency of anomalies gets down to 23.3%, which is almost the same as the number which we observe from the U.K. data.[19]

The third and the fourth columns are the regression results of the following linear regression model:

$$S_{t,rollover} = \alpha_0 + \alpha_1 J_{t-1} + \epsilon_t. \tag{7}$$

where "J_{t-1}" is the jackpot size rolled in from the $t-1$th issue. Regression (7) is only applied to the sales in the rollover samples, $S_{t,rollover}$. Sales in the regular draw are not taken into account since the jackpot size must start from 0 for all the regular draws. While the regression coefficient is positive and is significant, its explanatory power in terms of the coefficient of determination (R^2) is rather low as compared with the real data.

To get a closer look at the behavior of the rollover, Figure 5 shows the time series of the rollover frequency under different lottery tax scenarios.[20] It is quite evident to see that rollover frequency is positively affected by the tax rate. This is mainly because lottery participation is adversely affected by the lottery tax rate. When the lottery tax rate is high, the corresponding participation is low, hence it becomes much easier to observe the rollover of the jackpot. For example, when $\tau = 0.9$, the rollover frequency is almost as high as one. Figure 5 also shows that the rollover frequency declines over time when the lottery tax rate is not high, say, $\tau = 0.5, 0.3$ and 0.1. This is consistent with an increasing tendency to participate in the lottery which we found earlier (Figure 3).

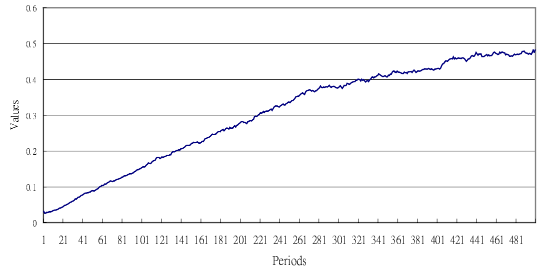

Figure 7: Time Series of the Degree of a Fair-Game Belief

Finally, Figure 6 shows the time series behavior of the rollover size. The series was drawn by pooling all the runs for all scenarios and was then further normalized by total income. Despite its constant fluctuations, a long declining tendency is quite evident. This result echoes well with the declining tendency of the rollover frequency (Figure 5). When rollover is less frequent, it becomes difficult for the jackpot to be accumulated consecutively, and hence its size with very likely start from zero, and it will be hard for it to get high.

Conscious Selection

Chen and Chie (2003) develop a metric to measure the degree of conscious selection, and this metric can give one an idea of how far or close the agent is to a *fair-game believer*. The metric d is briefly stated as follows:

$$d = \begin{cases} \binom{X-z}{x-z} / \binom{X}{x}, & if \;\; z \leq x, \\ \binom{z}{x} / \binom{X}{x}, & if \;\; z > x, \end{cases} \tag{8}$$

where z is the number of 1s appearing in $\vec{b}$. As can be easily shown, $0 \simeq d \leq 1$, and the higher the value of d, the higher the degree of fairness perceived by the agent.

Figure 7 is the time-series plot of the d metric, which is derived by taking the average over all simulation samples (25 runs over different τs). As expected, d starts from a very low number due to random initialization, but then there is a tendency for a fair-game belief to

[19] Ibid, Table 3. Of course, the problem is that the lottery tax rate in the U.K. is not as high 70%. So these two numbers do not fit nicely.

[20] Here, we compute the rollover frequency for each period based on the 25 runs with respect to different tax scenarios, and transform it into its 100-period moving-average counterpart.

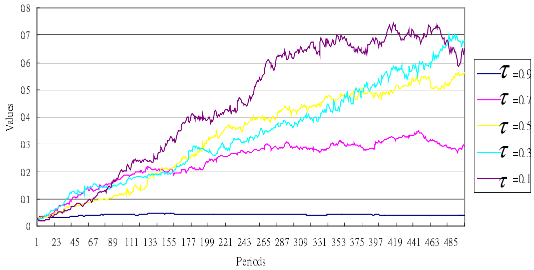

Figure 8: Time Series of the Degree of a Fair-Game Belief under Different Tax Rates

evolve.[21] Nevertheless, d does not converge enough to 1. Instead, it seems to settle around the 0.5 level, which is approximately equivalent to a z of 14. Therefore, a degree of conscious-selection behavior is *weakly* observed. This result is almost the same as the one observed in (1).

If we further examine the evolution of the fair-game belief by disaggregating it according to the tax scenarios as shown in Figure 8 , the upward tendency remains unchanged. However, we do notice the impact of τ. Generally speaking, d_t increases at a faster rate with the decrease in τ. When $\tau = 0.9$, there is almost no change in d_t. This result indicates that learning in terms of the fair-game belief becomes slow when the lottery tax rate is high, and it becomes extremely slow when $\tau = 0.9$. Basically, when people have already learned not to play the lottery, whether the game is fair or not becomes a secondary issue or even an irrelevant issue. Since the tax rate impacts participation, the speed of the emergence of the fair-game belief shown by Figure 8 is well explained.

Aversion to Regret

In this social learning framework, not only do agents learn from others, but their preference may also be interdependent. By watching the evolution of the aversion to regret (characterized by θ), we can actually see how this interdependent preference may emerge or disappear. We examine the values of θ for all of the 5,000 agents in the last period (period 500), and take an average from this sample. Let us call the average $\bar{\theta}$. Figure 9 is the box-whisker plot of $\bar{\theta}$ over the 25 runs. The line

Figure 9: Regret Coefficient and Lottery Tax Rate $\bar{\theta}$

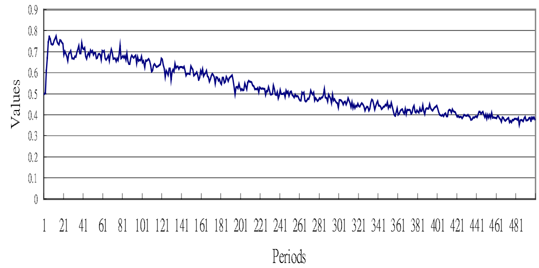

Figure 10: Time Series of the Regret Coefficient

inside the box shows the median of the 25 runs. If we just focus on the median, we see a relatively lower tendency to regret formed in these markets as opposed to that in (1), while they share a similar pattern regarding the impact of τ on the level of θ. For instance, τ tends to have a positive effect on the level of θ, while this effect is not significant when τ is low.

As we have learned from Figure 5, when τ is high, lottery participation becomes low, which makes the match more difficult and the rollover more easy. With the specification given in Equation (3), a high θ is then desirable for agents who do not gamble since they are very likely to gain additional satisfaction by observing that there is no match for the jackpot given the low participation rate. Hence, our agents are simply smart enough to learn to be more preference-dependent. On the other hand, when τ is low, lottery participation is high, and a rollover is less likely, since a high θ makes those non-participating agents more likely to suffer rather than gain. So agents in the market should learn to be less preference-interdependent. Furthermore, when lottery participation is high, e.g., if everybody participates, then the strategy parameter θ is no longer relevant, as we have seen regarding the d in Section , which may explain why the parameter θ becomes

[21] Here d is a reference number. Supposing the binary string is randomly generated, then on the average, we can expect that the frequency of 0 and 1 is half and half, which is 8 out of 16. That is initially $z = 8$, and

$$d_0 = \frac{\binom{8}{5}}{\binom{16}{5}} = 0.0128.$$

less sensitive to τ when it is low.[22]

To further confirm what we have just said, Figure 10 gives the time-series plot of θ. A strong declining tendency is observed here, meaning that agents become less and less interdependent. This declining tendency matches well with the increasing tendency of the lottery participation rate as shown in Figure 3.

Concluding Remarks

Agent-based modeling provides us with a complete and systematic treatment of human behavior in complex adaptive systems. This paper provides an illustration of this idea. The various kinds of interesting behavior emerging from the bottom up allow us to see how each piece of this development actually supports the others, and should be better studied together in a coherent body, rather than being treated independently or exogenously. Thus the social euphoria extensively observed in this lottery market co-evolves with an increasingly active lottery participation (Figures 3 and 4), which in turn co-evolves with declining rollover frequencies and rollover sizes (Figures 5 and 6) as well as a less interdependent preference (Figure 10).

As a by-product, agent-based modeling provides us with a tool to simulate evolution and learning, which enhances our study of bounded rationality. Apart from the co-evolutionary phenomena summarized above, *evidence of learning is prevalent in this model*, which includes belief updating and the associated decision on lottery participation (Figures 4 and 3), the emergence of the fair-game belief (Figure 7), and the development of the less interdependent preference (Figure 10). All these four figures exhibit a strong tendency, and the above-mentioned macro-dynamics are connected to these micro-behavior.

Having brought learning into our discussion, we are well aware of the lesson that *agent engineering matters*, or that learning or adaptive behavior can crucially change the final results. With this in mind, this paper considers a design that is different from that in (1). This new design modifies the behavioral foundation of agents. Originally, it was based on fuzzy decision rules, and now it is based on the expected-utility framework. *Does this change matter?* The answer is largely *no*. Most results we have from (1) remain robust to this change. The phenomenon of the Laffer curve, the absence of the halo effect, and the emergence of the fair-game belief remains unchanged, at least qualitatively. Even though the results remain unchanged, our understanding and interpretation of the same results may change because of the different add-on behavioral mechanisms. For example, the emergence of *social euphoria*, which is a main driving force to piece together the most interesting simulated results, is simply not available in (1), in which case a model of subjective belief is simply not there.

[22] It is interesting to see whether the theory of θ developed here can also help explain what we observe in (1). Actually, a similar finding is also observed in (1), where we see θ is again positively related to τ. The only difference is that the sensitivity of θ to τ becomes even weaker in (1), which seems to suggest that θ is not that relevant to a larger set of τ.

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