行政院國家科學委員會專題研究計畫 成果報告

非線性擾動下二維有界域半線性波方式爆炸解之穩定性研究（II）

研究成果報告(精簡版)

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共 同 主 持 人：謝宗翰、白仁德

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中 華 民 國 99 年 09 月 11 日

Stability of positive solutions for some semilinear wave equations under nonlinear perturbation near blow-up solutions in 2-space dimension

$$\square \text{ u } - \text{ u}^p + \lambda \text{u}^q = 0 \quad \text{(II)}$$

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Abstract In this research we treat the stability of positive solutions of some particular semilinear wave equations under nonlinear perturbation in bounded domain near blow-up solutions in 2-space dimension.

1. Introduction

Consider the initial value problem for the semilinear wave equation of the type

$$\square\, u + g\,(u) = 0 \quad in \quad [0, T) \times \mathbb{R}^2, \tag{0.1}$$

$$u\,(0, \cdot) = u_0,\ \dot{u}\,(0, \cdot) = u_1, \tag{0.2}$$

where $g : \mathbb{R} \to \mathbb{R}$is a real valued function, the initial data are given sufficiently smooth functions and $\square u := u_{tt} - \triangle u$, $\triangle$ is the Laplace operator. The linear case $g(u) = mu$, where m is a constant, corresponds to the classical Klein Gordon equation in relativistic particle physics; the constant m is interpreted as the mass and is assumed to be nonnegative generally. To model also nonlinear phenomena like quantization, in the 1950s equations of (0:1) type with nonlinearities like $g(u) = mu + u^3$; $m \geq 0$; were proposed as models in relativistic quantum mechanics with local interaction. Solutions could be considered as real or complex valued functions. In the latter case it was assumed that the nonlinearity commutes with the phase; that is, $g\left(e^{i\varphi}u\right) = \varepsilon e^{i\varphi} g\,(u)$ for $\varphi \in \mathbb{R}$ and that $g(0) = 0$. In this case, g may be expressed $g(u) = uf\left(|u|^2\right)$, which gives the study of equation (0:1) $[J]$. In the noncoercive case it is easy to construct solutions of (0.1) with smooth initial data that blow up in finite time; for instance, for any $\alpha > 0$ the function $u(t;x) = (1-t)^{-1/m}$ solves the equation $\Box u + \alpha(1+\alpha)u\,|u|^{2m} = 0; m \in \mathbb{N}$ and blows up at $t = 1$. Modifying the initial data off $\{x : |x| \leq 2\}$, say, we even posses a singular solution with C^{∞}-data having compact support.

In this study we want to deal with the stability of positive solutions for the semilinear wave equation

$$\Box\, u = u^p + \lambda u^q \text{ in } [0,T) \times \Omega, \Omega \subset \mathbb{R}^2 \tag{1.1}$$

with boundary value null and initial values $u(0,\cdot) = u_0(\cdot) \in H^2(\Omega) \cap H_0^1(\Omega)$ and $\dot{u}(0,\cdot) = u_1(\cdot) \in H_0^1(\Omega)$, where $p, q \in (1,\infty)$ and Ω is a bounded domain in $\mathbb{R}^2$.

We will use the following notations:

$$\cdot := \frac{\partial}{\partial t}, Du := (\dot{u}, \nabla u), \nabla u := \frac{\partial^2 u}{\partial^2 x} + \frac{\partial^2 u}{\partial^2 y},$$

$$a(t) := \int_\Omega u^2(t,x)\,dx, E_\lambda(t) := \int_\Omega \left(|Du|^2 - \frac{2}{p+1} u^{p+1} - \frac{2\lambda}{q+1} u^{q+1} \right)(t,x)\,dx.$$

For a Banach space X and $0 < T \leq \infty$ we set

$$C^k(0,T,X) = \text{Space of } C^k - \text{ functions} : [0,T) \to X,$$

$$H1 := C^1\left(0,T,H_0^1(\Omega)\right) \cap C^2\left(0,T,L^2(\Omega)\right).$$

The existence result to the equation (1.1) is proved $[Li\ 3]$ and the positive solution blows-up in finite time if $\lambda \geq 0$ $[Li\ 2]$, this means that the positive solutions for the semilinear wave equation

$$\Box\, u = u^p \text{ in } [0,T) \times \Omega, \tag{1.2}$$

$$u(0,\cdot) = u_0(\cdot) \in H^2(\Omega) \cap H_0^1(\Omega),$$

$$\dot{u}(0,\cdot) = u_1(\cdot) \in H_0^1(\Omega),$$

is stable under nonlinear perturbation λu^q providing $p > 1, q > 1,\ \lambda > 0$; but it is not clearly whether it is also true for any $p > 1, q > 1,\ \lambda < 0$? If so, we would want to estimate the blow-up time and the blow-up rate under such a situation.

It is also important to study the asymptotic behavior of the solution u_λ; the velocity and the rate of the approximation for λ approaches to zero.

Such questions are also not easy to answer even under the case for the ordinary differential equation

$$u'' = u^p\left(c + \lambda u'(t)^q\right), \tag{1.3}$$

$$u(0) = u_0, u'(0) = u_1,$$

$$p > 1, q > 1, c > 0, \lambda > 0.$$

We have studied the blow-up behavior of the solution for problem (1.3) and got some estimates on blow-up time and blow-up rate [Li4] but it is difficult to find the real blow-up time (life-span). Further literature could be fund in [S], [R], [W1] and [W2].

In this study we hope that our ideals used in [Li 4], [Li 5] can do help us dealing such problem (1.1) on our topics.

2. **Definition and Fundamental Lemma**

There are many definitions of the weak solutions of the initial-boundary problems of the wave equation, we use here as following.

Definition 2.1: *For $p > 1$, $u \in H1$ is called a positive weakly solution of equation* (1.1)*, if*

$$\int_0^t \int_\Omega \left(\dot{u}(r,x)\,\dot{\varphi}(r,x) + (u^p + \lambda u^q)(r,x)\,\varphi(r,x)\right) dx dr$$

$$= \int_0^t \int_\Omega \nabla u(r,x) \cdot \nabla \varphi(r,x)\, dx dr \ \forall \varphi \in H1$$

and

$$\int_0^t \int_\Omega u(r,x)\,\psi(r,x)\, dx dr \geq 0$$

for each positive $\psi \in C_0^\infty([0,T) \times \Omega)$.

Remark 2.2:

1) The definition 1.1 is resulted from the multiplying with φ to the equation (1.1) and integrating in Ω from 0 to t.

2) From the local Lipschitz functions $u^p + \lambda u^q, p > 1, q > 1$ the initial-boundary value problem (1.1) possesses a unique solution in $H1$ [Li1]. Hereafter we use the notations:

$$\frac{1}{C_\Omega} := \eta_1 = \sup\left\{\|u\|_2 \Big/ \left\|\frac{\partial u}{\partial x}\right\|_2 : u \in H_0^1(\Omega)\right\},$$

$$\lambda_q = \sup\left\{\|u\|_q \Big/ \left\|\frac{\partial u}{\partial x}\right\|_2 : u \in H_0^1(\Omega) \cap L_q(\Omega)\right\}, q > 1.$$

In this study we need the following lemmas

Lemma 2.3: *Suppose that $u \in H1$ is a weakly positive solution of* (1.1) with $E_\lambda(0) = 0$ for $p > 1, q > 1$, *then for $a(0) > 0$ we have:*

(i) *$a \in C^2(\mathbb{R}^+)$ and $E_\lambda(t) = E_\lambda(0) \quad \forall t \in [0,T)$.*

(ii) *$a'(t) > 0 \quad \forall t \in [0,T)$, provided $a'(0) > 0$.*

(iii) *$a'(t) > 0 \quad \forall t \in (0,T)$, if $a'(0) = 0$.*

(iv) *For $a'(0) < 0$, there exists a constant $t_0 > 0$ with*

$$a'(t) > 0 \quad \forall t > t_0$$

and $a'(t) = 0$.

Lemma 2.4: *Suppose that* u *is a positive weakly solution in* $H1$ *of equation* (1.1) *with* $u(0,\cdot) = 0 = \dot{u}(0,\cdot)$ in $L^2(\Omega)$. *For* $p > 1, q > 1, \lambda > 0$, *we have* $u \equiv 0$ *in* $H1$.

According to Lemma 2.4, we discuss the following theme

(3) $E_\lambda(0) = 0, a(0) > 0$ and $a'(0) \geq 0$ or $a'(0) < 0$.

(4) $E_\lambda(0) < 0, a(0) > 0$ and $a'(0) \geq 0$ or $a'(0) < 0$.

3. Estimates for the Life-Span

3.1. Estimates for the Life-Span of the Solutions of (1.1) under Null-Energy.

We study the case that $E_\lambda(0) = 0,\ p > 1, q > 1, \lambda > 0$ and divide it into two parts

(i) $a(0) > 0, a'(0) \geq 0$

and

(ii) $a(0) > 0, a'(0) < 0$.

Remark 3.. 1) The local existence and uniqueness of solutions of equation (1.1) in $H1$ are known [Li 2].

2) For $\lambda = 0$, $p > 1$ and $E_\lambda(0) = 0$, the life-span of the positive solution $u \in H1$ of equation (1.1) is bounded by

$$T \leq \alpha_1 := k_2^{-1} \sin^{-1}\left(\frac{k_2}{k_1 a^{\frac{p-1}{4}}(0)}\right)$$

with

$$k_1 := \frac{p-1}{4} \cdot a^{-\frac{p-1}{4}}(0)\sqrt{a'(0)\,a^{-2}(0) + 4C_\Omega^2},\ k_2 := \frac{p-1}{2} C_\Omega,$$
$$\frac{1}{C_\Omega} := \eta_1 = \sup\left\{\|u\|_2 / \|Du\|_2 : u \in H_0^1(\Omega)\right\}.$$

3.2. Estimates for the Life-Span of the Solutions of equation (1.1) under Negativ-Energy.

We use the following result and those argumentations of proof are not true for positive energy, so under positive energy we need another method to show the similar results.

Lemma 3: *Suppose that* $u \in H1$ *is a positive weakly solution of equation (1.1) with* $a(0) > 0$ *and* $E_\lambda(0) < 0$ *for* $\lambda = 0$. *Then*

(i) for $a'(0) \geq 0$, *we have* $a'(t) > 0\ \ \forall t > 0$.
(ii) for $a'(0) < 0$, *there exists a constant* $t_5 > 0$ *with*

$$a'(t) > 0\ \ \forall t > t_5,\ a'(t_5) = 0$$

and

$$t_5 \leq t_6 := \frac{-a'(0)}{(p-1)\left(\delta^2 - E_\lambda(0)\right)},$$

where δ *is the positive root of the equation*

$$\frac{2}{p+1}\lambda_{p+1}^{p+1} \cdot r^{p+1} - r^2 + E_\lambda(0) = 0.$$

4. **Stability of positive solutions of equation (1.1) near blow-up solutions under Negativ-Energy**

In this study we use our ideals used in $[Li\ 2]$, $[Li\ 4]$, $[Li\ 5]$ and $[Li\ 7]$to deal such problem (1.1) on our topics under negative energy and obtain the following results:

Theorem 4.1: *Suppose that* $u_\lambda \in H1$ *is a weakly positive solution of* (1.1) with $E_\lambda(0) \leq 0$ for $p > 1, q > 1,$ *then for* $a(0) > 0$ *we have*:

The equation (1.1) *is stable for* $\lambda \to 0^+$; *this means that weakly positive solution* u_λ *of* (SL) *blows up in finite time for* $\lambda \to 0^+$.

Theorem 4.2: *Suppose that* $u_\lambda \in H1$ *is a weakly positive solution of* (1.1) with $E_\lambda(0) \leq 0$ for $p > 1, q > 1,$ *then for* $a(0) > 0$ *we have*:

The equation (1.1) *is stable for* $p > q$, $\lambda \to 0^-$; *this means that weakly positive solution* u_λ *of* (1.1) *blows up in finite time for* $p > q$, $\lambda \to 0^-$.

Theorem 4.3: *Suppose that* $u_\lambda \in H1$ *is a weakly positive solution of* (1.1) with $E_\lambda(0) \leq 0$ for $p > 1, q > 1,$ *then for* $a(0) > 0$ *we have*:

The equation (1.1) *is unstable for* $p < q$, $\lambda \to 0^-$; *this means that some weakly positive solution* u_λ *of* (1.1) *blow up in finite time for* $p < q$, $\lambda \to 0^-$; *but also there were some global weakly positive solution* u_λ *of* (1.1) *for* $p < q$, $\lambda \to 0^-$.

Remark:

The decade rate of the difference of life-spans T_λ of u_λ and T of u, can not be estimated very well for $\lambda \to 0$; thus it will be a good topic on asymptotic behavior near the blow-up solutions.

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報告

十月十五日討論 波茲曼方程在實際上應用於其它領域的可行性

1. 建立個體經濟新模型之可行性
2. 建立總體經濟新模型之可行性
3. 建立人口模型之可行性
4. 稀薄流新的思考方向 建立新模型之可行性

十月十六日討論 波茲曼方程中碰撞項於實際應用在其它領域時
進行實質分類的可行性

1. 經濟行爲如個經 總經或上市公司間之交互持股 其效益問題建模時 其碰撞項的分類明顯重要
2. 人口問題亦應可利用波茲曼方程的概念來建模 於建模時其碰撞項的分類 亦格外重要
3. 若可建立稀薄流新模型時其碰撞項的分類 格外重要 尤其在經費的支出上

十月十七日 因楊彤教授赴廣州大學參加學術研討會 所以無法共同討論細節 相約于十一月份再進行細節上的討論 併討論楊彤教授訪問政大應數系的可能性與時間

十月十八日 整理行裝準備返台

Report on visiting Chair Professor Tong Yang
(Professor of Institute of Technology Vice-President)
Department of Mathematics, City University of Hong Kong

October 15 Discussion about the feasibility of the applications of Boltzmann equations practically, to other areas:

1. To establish the feasibility of a new model of Microeconomics
2. The establishment of the feasibility of a new model for Macroeconomics
3. To establish the feasibility of the population model
4. Possibilities to explore a new direction on Rarefied flow and the feasibility establishing a new model

October 16 to discuss of the substantive classification of Boltzmann equations in the collision term of the practical application in other research fields

1. Economic behavior, such as the listed company or by the interaction between the holdings of its benefits

   Problem modeling the classification of items of its obvious importance collisions

2. The population problem should be able to use the concept of Boltzmann equation to model in the modeling

   Its impact is particularly important to the classification of items

3. If it is able to establish the thin stream of new models of its impact is particularly important, particularly the classification of items

   The expenditure of funds,

October 17 by Professor Yang Tong went to the Guangzhou University Symposium on it is not possible to participate in the common

Meet in November to discuss the details further discussion of the details and discuss the Tong Yang

Visit the Department of National Chengchi University professor to be a few possibilities and the time

October 18 packing and return to Taiwan

訪香港城市大學數學系

楊彤教授(講座教授 理工學院副院長)

報告

十一月十九日討論 波茲曼方程在實際上應用於其它相關領域的可行性

1. 建立個體經濟之分子運動非線性模型之可行性
2. 建立總體經濟之分子運動非線性模型之可行性
3. 建立非線性人口模型之可行性
4. 稀薄流新的思考方向 建立二次非線性模型之可行性

十一月二十日討論 波茲曼方程中碰撞項於實際應用在其它領域時
進行二次非線型逼近的可行性

1. 經濟行爲之效益問題建模時 以二次非線型分子運動逼近明顯重要
2. 人口問題亦可以波茲曼方程概念建模 其碰撞項之二次非線型逼近亦格外重要
3. 建立稀薄流時碰撞項二次非線性模型

十一月二十一日 相約于二 O 一 O 年六月楊彤教授訪問政大應數系
十一月二十二日 整理行裝準備返台

無研發成果推廣資料

98 年度專題研究計畫研究成果彙整表

計畫主持人：李明融	計畫編號：98-2115-M-004-004-
計畫名稱：非線性擾動下二維有界域半線性波方式爆炸解之穩定性研究（II）	

成果項目			量化：實際已達成數（被接受或已發表）	量化：預期總達成數(含實際已達成數)	量化：本計畫實際貢獻百分比	單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）
國內	論文著作	期刊論文	1	1	100%	篇	Parabolic method in o.d.e , 2010 accepted by Taiwanese J. Mathematics)
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		
國外	論文著作	期刊論文	1	1	100%	篇	submitted to a sci journal
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%	章/本	
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	1	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	1	0	100%		

其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)	無

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

■達成目標

□未達成目標（請說明，以 100 字為限）

□實驗失敗

□因故實驗中斷

□其他原因

說明：

2. 研究成果在學術期刊發表或申請專利等情形：

論文：■已發表 □未發表之文稿 □撰寫中 □無

專利：□已獲得 □申請中 ■無

技轉：□已技轉 □洽談中 ■無

其他：（以 100 字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限）

’’parabolic method in o.d.e.’’ 2010 已被接受在 Taiwanese Journal of Mathematics(sci)

旨於臺灣人口成長問題與特性

另一文’’Mathematical model of enterprise competitiveness and performance’’

已投 sci 期刊　正審核中

以上二文皆引此報告之法 究研對象特質成文